ASSISTING GRADES K-4 WITH MATHEMATICS IN THE CLASSROOM

Assisting Grades K-4 with Mathematics in the Classroom

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Academy Introduction

This Academy is designed to provide paraeducators with the skills and knowledge needed to assist students, kindergarten through grade 4, with mathematics skills taught in the classroom. The course content is based on and adapted from standards recommended by the National Council of Teachers of Mathematics. It includes the specific skill-building areas of number sense, computational techniques, algebraic thinking, geometry, measurement, data, and probability as they apply to younger learners in elementary schools.

Module A
Instructor's Guide

Assisting Grades K-4 with Mathematics in the Classroom

Module A: Mathematical Literacy

A. Energizer: Introductions

Provide an energizer that will help participants become familiar with each other and with the instructor and increase the likelihood of greater class participation. If possible, create an anticipation and excitement about working with mathematics.

Note to Instructor: This is an appropriate time to review the need for well-organized notebooks. Each participant should bring a 3-ring binder in which to keep handouts, personal notes, and materials used in the class. If the class is offered for credit, reiterate to the participants that they will be taking an **assessment at the end of the class that will be an "open book test."** At this point also share the **grading rubric** handout (**GR**) to inform the participants about other requirements they must meet in order to receive a grade. Explain that the more highly organized and detailed their personal notebooks are, the more comfortable they will be with the assessment, and the more likely they will be to do well on the assessment. It is recommended that you bring a 3-hole punch to class for participants use and to make sure that all handouts are run on 3-hole paper.

B. Focus Activity: Math Journal Responses

Paraeducators will participate in an activity that will help them analyze their personal view of mathematics and their perceived knowledge and abilities. This *ungraded* introduction activity is done prior to the course introduction because the activity will be biased once participants feel that the correct answers lie in the course goals being presented.

B.1 Steps

- Organize the materials needed. **You will need to provide markers, crayons, colored pencils, glue, scissors, etc., for this activity.**
- Emphasize that this activity will not be graded and will not be viewed or reviewed by other members of the group.
- Use the **First Thoughts about Math** handout **(H1).** Provide a handout to each member of the class. Emphasize to participants that if they are using a 3-ring binder to organize their materials, they need to include labeled sections in which to keep specific entries.
- Ask participants to respond to the questions on the **First Thoughts about Math** handout. They can respond to the questions using writing, drawings, or any other format they are comfortable with.

First Thoughts about Math

1. How would you define mathematics?
2. Who uses mathematics?
3. How would you describe/rate you mathematical ability?

- Use the **Response Chart 1** and **Response Chart 2 (T1/T2)** transparencies for the first two questions. After participants have completed the activity (attempt to keep the time for this to approximately 10 minutes), ask them to share their thoughts with the rest of the class. Record their responses on the transparencies.
- Do not spend a lot of time discussing or debriefing the responses to these questions. They will be revisited in the first goal of the module.
- Share with the group that their responses are typical of those of others, both students and adults.

C. Lecture: Academy Introduction

Using the **Academy Goals** transparency and handout **(T3/H2),** introduce and review the contents of the Academy.

Review the Academy goals using the following lecture information.

This Academy is designed to provide paraeducators with the skills and knowledge necessary to assist students, kindergarten through fourth grade, with mathematics skills taught in the classroom. The course content is based on and adapted from the standards and expectations recommended by the National Council of Teachers of Mathematics. It includes developing the concept of mathematical literacy and specific skill building in number sense, computational techniques, algebraic thinking, spatial reasoning, measurement, data organization, and probability as they apply to early-elementary learners.

Two main ideas flow through the structure of this Academy. The first main idea is to build and strengthen the mathematics skills of the paraeducator. While many paraeducators know some or most of the mathematics presented in this Academy, it is important that they view the activities with a deeper level of understanding. To be able to explain concepts to a child requires that the paraeducator view activities and concepts through the eyes of the student. Have the paraeducators reflect on their personal experiences of working with students to explore holes in their understanding and possibly find solutions to how to help with weak areas. It is important for the paraeducators to look for patterns and connections throughout the Academy in order to provide comprehensive assistance to their students.

The second continuous idea in this Academy is the importance of fully comprehending the activities. The activities were chosen as samples of activities that paraeducators could do with students. Many of them contain titles that may feel quite elementary for this course. Studies show that students retain information better from activities that can be referred to by a name such as “Mail It” or “Clear the Boards” rather than just doing a worksheet. As discussions occur in the course, reference the material by referring back to the related activity. This helps make connections between concepts.

Module A: Mathematical Literacy
The paraeducator will:
1. Identify common misconceptions about mathematics
2. Identify the role of communication in mathematical literacy development
3. Identify the goal of problem-solving and its development in the classroom
4. Compare and contrast mathematical literacy and language/reading/writing (literacy) development

Module B: Patterns
The paraeducator will:
1. Use concrete materials to aid pattern recognition and generalization
2. Describe patterns and other relationships to interpret data using tables and graphs
3. Employ strategies of problem-solving to make predictions and determine the likelihood of an event
4. Relate basic patterns to algebraic concept development

Module C: Number Representation and Manipulation
The paraeducator will:
1. Use multiple models to develop understanding of place value and the base-10 number system
2. Use patterns to explore algorithms for basic mathematical operations
3. Understand the meaning, effects, and relationships of the basic mathematical operations
4. Define and communicate scenarios for appropriate use of basic operations (applications, money, time, etc.)

Module D: Equivalency and Number Comparison
The paraeducator will:
1. Use a variety of concrete materials to develop meanings for commonly used fractions and decimals for sets and wholes
2. Demonstrate equivalent forms of the same number through the use of models, drawings, and other strategies
3. Compare numbers as equal, greater than and less than, using a variety of strategies

Module E: Spatial Development and Measurement
The paraeducator will:
1. Recognize and explore 2-D geometric shapes by their attributes (specific quadrilaterals, triangle, and circle; symmetry, diagonals, etc.)
2. Recognize and explore 3-D geometric shapes by their attributes (cube, cylinder, cone, and pyramid)
3. Solve problems using geometric relationships and spatial reasoning (e.g., coordinate geometry, congruence, similarity)
4. Identify angle types
5. Use both standard and non-standard measurement for perimeter and area

Goal 1: Identify common misconceptions about mathematics.

1.1 Discussion: Common Misconceptions about Mathematics

Paraeducators will define some of the major misconceptions about "mathematics" that they will encounter while working with students and their parents.

1.1.1 Steps

- Direct participants to return to the **First Thoughts about Math** handout **(H1)** used in the focus activity while you return to the **Response Chart 1** and **Response Chart 2** transparencies **(T1/T2).** Briefly review the recorded answers/responses. Ask participants to review their own private responses. Have them compare their responses to the ones listed in the next step.
- Use the **Question 1: Common Responses** transparency **(T4).** Discuss the listed common answers and misconceptions about mathematics from both children and adults. Many other answers are possible; these are only a few that will help spark discussion:
  Question 1: Common Responses
  ▲ Word problems (problem-solving)
  ▲ Numbers
  ▲ Rules
  ▲ Problems
  ▲ Memorization
  ▲ Skills
  ▲ Drill
  ▲ Homework
  ▲ Not fun
  ▲ Too hard
- Use the **Question 2: Common Responses** transparency **(T5).** Discuss the listed common answers and misconceptions about mathematics from both children and adults. Many other answers are possible; these are only a few that will help spark discussion:
  Question 2: Common Responses
  ▲ Scientists (doctors, teachers, etc.)
  ▲ Smart people
  ▲ Parents (adults)
  ▲ People in jobs (stores, waiters/waitresses, etc.)
  ▲ No one – it is just for school *(some young students believe this)*
- Use the **Response Chart 3** transparency **(T6).** Brainstorm with attendees why they might perceive mathematics as reviewed in **Response Chart 2**. Record their answers. Possible responses to look for include the following:
  ▲ Only certain people can do math
  ▲ Not successful
  ▲ Too much vocabulary (language problems)
  ▲ Don't see math in their everyday life like other literacy skills
  ▲ Socioeconomic-cultural background
  ▲ Gender differences

Note: Current studies show that girls are turned off by mathematics by fourth grade.

1.2 Lecture: Defining Mathematics

Return to the transparency **Question 1: Common Responses (T4).** Ask attendees which responses on the list represent *arithmetic*. Check off the responses that represent ***arithmetic*** (all but the last two). Discuss with participants that common misconceptions are often the result of confusing definitions. Use the **Arithmetic** transparency **(T7).**

Arithmetic

Arithmetic – calculations involving predefined rules

Arithmetic is an important part of mathematics but does not encompass all of what is included in a definition of mathematics. *Arithmetic skills* such as adding, subtracting, using fractions, algebra, etc., are all a major reason why students/adults say they dislike *mathematics*.

Note: Remind participants that many of the students paraeducators work with may have low *arithmetic* skills, but not necessarily low *mathematics* skills.

Explain that this statement may cause some confusion. Another way to look at this would be to ask for a show of hands for the following:

"How many of you enjoy the following?"

- Doing puzzles
- Art/drawing
- Landscaping
- Cooking
- Decorating

Explain that all of these activities are examples of mathematic activities. They all involve skills that mathematicians use. In the following modules we will examine this much more closely. Use the **Mathematics is ...** handout and transparency **(H3/T8).** As you review the points of the transparency, explain how each will be covered in the Academy. Discuss what Reys, Suydam, and Lindquist (1992) say about how to define mathematics.

Mathematics is …

Mathematics is …

1. a study of patterns and relationships (*Module B*)
2. a way of thinking (*all modules*)
3. an art (*involves creativity – not just rules*)

4. a language (*we will spend more time on this*)
5. a tool (*used in almost everything we do*)

– Reys, Suydam, and Lindquist (1992)

Remind participants that in order to understand mathematics as being more than arithmetic, they must recognize that math exists in almost every aspect of their lives, from when they get out of bed in the morning (telling time on a clock) to choosing their clothes for the next day (probability) in the evening.

Goal 2: Identify the role of communication in mathematical literacy development.

2.1 Lecture: Math as a Language

Part of understanding the definition of mathematics includes seeing math as a language. For example, students have oral language before they can read. Those skills begin to develop early in life. Children's language skills help them to develop further literacy skills, which include word recognition, reading, and comprehension skills. They also grow up seeing and hearing people around them using communication, reading, and writing skills. They are *obviously and observably* learning language, how to communicate, further extending the skills of reading and writing.

Learning mathematics is not such an obvious process. Young students do not have obvious early experiences with mathematics; they, and the adults around them, often do not recognize mathematics in use around them. Therefore, when mathematics, commonly arithmetic, is introduced, it often appears and is practiced as something outside of their regular experiences. For students whose first language is not English, mathematics appears to be another foreign concept to be learned.

In reality, mathematics functions along similar lines as language. That is, mathematics has its own rules, terms, and symbols that require the same practice as learning a language. Treating math as a language changes the methods by which we teach it.

Using the **Learning a Second Language** transparency **(T9)**, ask attendees to share their responses regarding what they believe is required to learn a second language. Record their responses on the transparency. Possible responses include:

- Exposure
- Practice/use
- Taking classes
- Reading and writing it
- Immersion in the culture representing the language

Now make the connection between learning a foreign language and learning mathematics. There are many similarities. Use the **Math as a Language** handout and transparency **(H4/T10)** to help explain math as a language.

Math as a Language

To learn the math language students/learners must:

1. Read it
2. Write it
3. Speak it
4. Do it

Read it –Implies more than just reading directions. Reading involves recognizing vocabulary, forming and interpreting math sentences, and following other students' written ideas (e.g., 3 + 2 = 5). As a sentence this problem could read:

"Put together 3 cars and 2 more cars is (gives) how many cars?"

"It gives me 5 cars."

Write it – Implies more than homework problems. This means explaining a mathematical problem with drawings, words, labels, and vocabulary. This reinforces the reading skill.

Speak it – Teachers should not be the only ones speaking the language. Students need practice with the language. They need to hear themselves, hear models (teachers or other adults), and hear their peers. This requires students to discuss math or participate in mathematical discussions. This includes having children talk out their work before ever putting pencil to paper.

Do it – This requires individual practice. It means trying out the language skills in homework practice, in group conversations, or in a reflective journal. Putting all of the prior skills together allows the student to use the language.

These four ideas include many literacy skills: reading, writing, communicating, listening, and speaking. These skills are not traditionally thought of as mathematics skills. If we attempt to link language skills and mathematics to help students become more successful at mathematics, we must be open to changing our practices and beliefs and try more than just the traditional methods that are typically offered. We are talking about a change in philosophy. It is important to recognize the factors of this philosophy and why they are important.

Discuss the importance of this change of philosophy for mathematics.

- Students can link mathematics with successful prior learning.
- Students can build on language skills they may already possess and add to their language base with the language of mathematics.
- Students can better communicate their understandings to both the teacher and their peers.
- Students can begin to see the mathematics process as a work in progress when they hear and participate in the process.

This change in philosophy emphasizes the role of communication in mathematical literacy development. Language is an important element in literacy. The purpose of language is to develop clear communication of ideas and processes. *This is as important in mathematics as it is in everyday social communication.*

2.2 Activity: Creating a Math Journal

Paraeducators will continue creating their math journals as a model for use with students. This portion of the journal will focus and build on the need for math communication.

2.2.1 Steps

- Using the **Math Journal** handout and transparency **(H5/T11)**, introduce the math journal. These handouts can serve as the dividers for the journal.
- Explain that this journal will consist of three sections. Each participant is expected to provide his or her own paper for taking notes within each section. Handouts should be filed appropriately within the following organization:
  - ▲ reflections
  - ▲ problem-solving
  - ▲ reference
- Discuss the first section, *reflections*, including the following information about the paraeducators' use of the journaling process.
  - ▲ Reflections from students often provide new insight into student understanding, often more so than homework and problem performance.
  - ▲ Journals help students practice communication for mathematics.
  - ▲ The **First Thoughts** handout **(H1)** should be under this heading.
- The second section is for recording in-class *problem-solving* work.
  - ▲ This keeps students' work together in one place and makes it easy to go back and look at strategies and processes. When paraeducators begin carrying over some of the strategies and skills they are learning in this Academy with their students, they will frequently refer to this section. Good organization provides easy access and increases the likelihood that the paraeducators will make good use of what they are learning.
- The last section of the journal is used as a *reference* tool.
  - ▲ Each working period, students should be asked to put unfamiliar or new concepts or vocabulary in their reference area.
  - ▲ Students should receive help with the proper spelling of the vocabulary words but they should define the word/concept in their own words and use pictures where necessary.
- The journal is designed to be something the students will use on their own and should be individually organized for best personal use.
- The journal supports paraeducators when no teacher is present and helps them be successful.
- Use of a math journal is a particularly important practice when working with students who are second-language learners; math may seem difficult and overwhelming to these students because of the numerous unique terms

used. The paraeducator can keep a list of new learning for each student and be better prepared to seek assistance with problem teaching areas.
- A journal provides the opportunity for the necessary repetition of ideas.

Goal 3: Identify the goal of problem-solving and its development in the classroom.

3.1 Lecture: Define Problem-Solving

Use the **Problem-Solving: A Definition** handout and transparency **(H6/T12).**

The National Council of Teachers of Mathematics (NCTM) (2000) defines problem-solving as follows:

Problem-Solving: A Definition

> Problem-solving means engaging in a task for which the solution method is not known in advance.
>
> In order to find a solution, students must draw on their knowledge, and through this process, they often develop new mathematical understandings.
>
> Solving problems is not only a goal of learning mathematics, but also a major *means* of learning mathematics.
>
> Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should be encouraged to reflect on their thinking.
>
> – NCTM (2000)

While this is a lengthy definition, it includes important points about problem-solving.
- Problem-solving should not be a separate unit of learning.
- Problem-solving should not be confused with "word problems." Many word problems rephrase an arithmetic problem.
- Good application problems allow for problem-solving beyond the use of a simple algorithm (rule).
- Problem-solving is the adhesive that holds the math curriculum together.

Children are naturally curious and are typically very flexible when they encounter new learning. Most often they do not fear new challenges and like the excitement of solving a problem or challenge. They enjoy experimenting with new methods of problem-solving and do not fear mistakes or failure. The need to solve a problem generates the need for acquisition of the skills. *Problems must be posed where solutions are not immediately obvious*. This encourages children to take a risk and strive to find a solution.

3.2 Activity: Problem-Solving Practice

Class members will participate in an activity requiring practice of personal problem-solving skills and examination of the problem-solving skills of others.

3.2.1 Steps

- Divide the class into small groups of 4-6.
- Using the **Problem-Solving** transparency **(T13)**, pose the following question to the group:

  Problem-Solving:
  "I have five coins in my pocket, some dimes, nickels, and pennies. I pull three coins out. How much money can I have in my hand? What is the greatest amount of money? What is least amount of money?"

- Direct the groups to work through the problem. Ask someone in each group to take notes to be shared with the entire class. Their notes should include how the group came to answers or conclusions; what was their thinking? Include the thoughts in the Problem-Solving section of the journal.
- Share group answers with the entire class.

3.3 Lecture: Problem-Solving and the Coin Activity

This problem presents an interesting question to children as money is a common part of their experience. They will soon find that there are several answers to the problem. This often surprises young students. Groups may choose to create rules such as restricting the starting number of coins or restricting the numbers of individual coins (e.g., 3 dimes, 1 nickel, and 1 penny). Others will want to use the ambiguity to find many solutions. Allowing students to manipulate problems shows them that mathematics is flexible. Allowing students to generate their own questions or variations also provides opportunities to make problem-solving personal. While some questions may not be mathematically appropriate due to students' grade level, students who can make valid connections are developing an understanding of mathematics.

As a group, discuss what math skills/concepts are necessary to solve the problem. After soliciting responses from the group, use the **Skills and Concepts** transcript **(T14)** to review or to provide information.

Skills and Concepts
- Coin recognition
- Coin value recognition
- Adding money amounts
- Organizing data
- Decimal knowledge (depending on grade level)
- Greatest, least (magnitude)

It is important for paraeducators to understand that as students tackle new problems, it is helpful if they have a plan. This is an area in which a paraeducator can be very helpful to students (*How to Solve It*. Doubleday & Co., 1957). George Polya produced a quick guide to help students become good problem-solvers. He offers the following suggestions. Use **Problem-Solving with a Plan** transparency and handout **(T15/H7)**.

Problem-Solving with a Plan

Steps for Problem-Solving

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

Understand the problem

- Rephrase the problem in your own words
- What are you trying to find or do?
- What are the unknowns?

Devise a plan

Throughout any math course, students need tools. Tools are not just rules. Tools involve strategies. While certain names are presented here, they are not standard, and students should not focus on memorizing them. They are simply ways to refer to processes that can be useful in solving problems. This is only a partial list for discussion:

Problem-Solving Strategies

- Look for a pattern
- Create a simpler problem
- Make a table
- Draw a diagram
- Write an equation
- Guess and check
- Work backward
- Identify a subgoal for the problem (involves breaking a problem down)

As students develop a plan, they often want to get to work rather than finishing their plan. For young children, talking about a plan and recording it is often a good place to start developing problem-solving skills.

Carry out the plan

- Implement the strategy
- Perform any necessary computations
- Keep record of your work

Look back

- Check results
- Revise the plan
- Make connections to other problems
- Try again if necessary

Looking back is often the most difficult step for students. They assume that when they get to the end, it is over. Many students frustrate easily if they get the problem incorrect the first time. Part of problem-solving is learning to revise and try again. This supports the old adage, "If at first you don't succeed, try, try again."

This method also follows Bloom's Taxonomy very closely. That is, learners go through the stages of knowledge and comprehension as they come to understand the problem. Then they use application skills as they devise a plan to solve the problem. Next, they carry out the plan and begin to analyze the plan for success. Last they look back at their activity and synthesize and evaluate what they have learned.

Goal 4: Compare and contrast mathematical literacy and language/reading/writing (literacy) development.

4.1 Activity: Compare and Contrast

Paraeducators will compare and contrast mathematical literacy and language/reading/writing (literacy) development.

4.1.1 Steps

- Using the **Comparing and Contrasting Reading and Math Literacy** handout **(H8),** direct paraeducators to work in small groups.
- Ask them to respond to the question, "What skills/expectations do you think a student has to demonstrate in order to have the language/reading/writing (literacy) skills needed in grades K-4?"
- Direct them to assign a note taker and representative from each group to represent their responses to the class during the large-group followup portion of this activity.
- After groups have had time to meet and share, ask them to share their responses with the entire class. Record their responses on the **Comparing and Contrasting Reading and Math Literacy** transparency **(T16).** If the recorded responses do not include the items on the list below, add pertinent ones to the transparency class list.

Comparing and Contrasting Math and Reading Literacy

Reading
- ▲ Letter recognition
- ▲ Letter sounds
- ▲ Word recognition
- ▲ Word comprehension
- ▲ Sentence production (rules)
- ▲ Sentence comprehension
- ▲ Paragraph summarization
- ▲ Order of events
- ▲ Prediction

Language
- ▲ Letter sounds
- ▲ Appropriate pronunciation
- ▲ Ability to communicate ideas
- ▲ Formulate questions and responses

Writing
- ▲ Letter formation
- ▲ Sentence production
- ▲ Ability to communicate ideas

- Provide class members with the **Defining Literacy** handout and transparency (**H9/T17**).

Defining Literacy

Literacy	**Mathematical Literacy**
Fundamentals (letters, sounds, etc.)	Fundamentals (numbers, symbols, etc.)
Rules (punctuation, spelling, etc.)	Rules (algorithms, order of operations, etc.)
Sentence production to represent ideas	Mathematical sentences (equations)
Recognizing practical uses outside the classroom (magazines, signs, conversations, etc.)	Recognizing practical uses outside the classroom (money, time, distance, shapes, etc.)
Comprehension	Ability to explain a problem or process
Prediction	Patterns and problem-solving
Communication	Communication

Ask the small groups to reconvene and discuss how the skills they listed previously fall into the categories on the handout: fundamentals, rules, sentence production, practical uses, comprehension, prediction, and communication. Direct the groups to discuss the same categories of mathematical literacy and compare the fundamental concepts that are also true in this area.

4.2 Lecture/Conclusion: Defining Mathematical Literacy

The term “mathematical literacy” implies that mathematics is not a static, one-sided concept. This is the new philosophy promoted in many current elementary mathematics programs.

The handout **Defining Literacy (H9)** contains only a few solid comparisons for the development of both types of literacy. Success in mathematics requires the same expectations that are required for reading, writing, and language skills.

While individual skills may not line up exactly, foundational concepts tend to run parallel with the new definition of mathematics. Paraeducators may need help in discussing and reorganizing their thinking. This concept will be reinforced and revisited throughout the Academy.

Module A Handouts

First Thoughts about Math

Please respond to the following journal questions. Feel free to respond using writing, drawings or any other format with which you are comfortable.

1. How would you define mathematics?

2. Who uses mathematics?

3. How would you describe/rate you mathematical ability?

Academy Goals

Assisting Grades K-4 with Mathematics in the Classroom

Module A: Mathematical Literacy
The paraeducator will:
1. Identify common misconceptions about mathematics
2. Identify the role of communication in mathematical literacy development
3. Identify the goal of problem-solving and its development in the classroom
4. Compare and contrast mathematical literacy and language/reading/writing (literacy) development

Module B: Patterns
The paraeducator will:
1. Use concrete materials to aid pattern recognition and generalization
2. Describe patterns and other relationships to interpret data using tables and graphs
3. Employ strategies of problem-solving to make predictions and determine the likelihood of an event
4. Relate basic patterns to algebraic concept development

Module C: Number Representation and Manipulation
The paraeducator will:
1. Use multiple models to develop understanding of place value and the base-ten number system
2. Use patterns to explore algorithms for basic mathematical operations
3. Understand the meaning, effects, and relationships of the basic mathematical operations
4. Define and communicate scenarios for appropriate use of basic operations (applications, money, time, etc.)

Module D: Equivalency and Number Comparison
The paraeducator will:
1. Use as a variety of concrete materials to develop meanings for commonly used fractions and decimals for sets and wholes
2. Demonstrate equivalent forms of the same number through the use of models, drawings, and other strategies
3. Compare numbers as equal, greater than and less than, using a variety of strategies

Module E: Spatial Development and Measurement
The paraeducator will:
1. Recognize and explore 2-D geometric shapes by their attributes (specific quadrilaterals, triangle, and circle; symmetry, diagonals, etc.)
2. Recognize and explore 3-D geometric shapes by their attributes (cube, cylinder, cone, and pyramid)
3. Solve problems using geometric relationships and spatial reasoning (e.g., coordinate geometry, congruence, similarity)
4. Identify angle types
5. Use both standard and non-standard measurement for perimeter and area

Mathematics is …

1. a study of patterns and relationships
2. a way of thinking
3. an art
4. a language
5. a tool

– Reys, Suydam, and Lindquist (1992)

Math as a Language

Math as a Language

To learn the math language students/learners must:

1. Read it
2. Write it
3. Speak it
4. Do it

Read it – Implies more than just reading directions. Reading involves recognizing vocabulary, forming and interpreting math sentences, and following other students' written ideas (e.g., 3 + 2 = 5). As a sentence this problem could read:

"Put together 3 cars and 2 more cars is (gives) how many cars?"
"It gives me 5 cars."

Write it – Implies more than homework problems. This means explaining a mathematical problem with drawings, words, labels, and vocabulary. This reinforces the reading skill.

Speak it – Teachers should not be the only ones speaking the language. Students need practice with the language. They need to hear themselves, hear models (teachers or other adults), and hear their peers. This requires students to discuss math or participate in mathematical discussions. This includes having children talk out their work before ever putting pencil to paper.

Do it – This requires individual practice. It means trying out the language skills in homework practice, in group conversations, or in a reflective journal. Putting all of the prior skills together allows the student to use the language.

Math Journal

Reflections

- I will reflect on my personal responses and insights into student understanding.
- I will reflect on my thoughts and insights about my own learning.
- I will reflect on my “ah-ha” moments—the “light bulb” just went on for me. Helps me remember what I learned and how I learned it.
- Writing about the processes I am learning will help me communicate my learning with others.

Problem-Solving

- It’s easier to remember how I solved a problem if I can review my own process.
- Keeps my work samples organized, and I can find and reference them easily.
- As I carry over my skills with students, I will be able to review my own work to help me clarify my own process.

Reference

- I will record unfamiliar concepts.
- I will record definitions and other language that will be helpful later.
- I can keep a list of new learning for each student.
- I can be better prepared to seek assistance with problem teaching areas.

Problem-Solving:
A Definition

Problem-solving means engaging in a task for which the solution method is not known in advance.

In order to find a solution, students must draw on their knowledge and, through this process, they often develop new mathematical understandings.

Solving problems is not only a goal of learning mathematics, but also a major *means* of learning mathematics.

Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should be encouraged to reflect on their thinking.

– NCTM (2000)

Problem-Solving with a Plan

Steps for Problem-Solving:

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Looking back

Understand the problem

- Rephrase the problem in your own words
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Carry out the plan

- Implement the strategy
- Perform any necessary computations
- Keep a record of your work

Look back

- Check results
- Revise the plan
- Make connections to other problems
- Try again if necessary

Comparing and Contrasting Reading and Math Literacy

Instructions: Please record personal responses, small-group responses, and responses from whole-class discussion in the columns below.

> "What skills/expectations do you think a student has to demonstrate in order to have the appropriate language/reading/writing (literacy) skills needed in grades K-4?"

Reading	Language	Writing

Defining Literacy

Reading Literacy	Mathematical Literacy
Fundamentals	Fundamentals
Rules	Rules
Sentence production to represent ideas	Mathematical sentences
Recognizing practical uses outside the classroom	Recognizing practical uses outside the classroom
Comprehension	Ability to explain a problem or process
Prediction	Patterns and problem-solving
Communication	Communication

Module A
Transparencies

Response Chart 1

Define Mathematics

-
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-
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-
-

Response Chart 2

Who Uses Mathematics?

-
-
-
-
-
-
-
-
-

Assisting Grades K-4 with Mathematics in the Classroom

Module A: Mathematical Literacy
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1. Identify common misconceptions about mathematics
2. Identify the role of communication in mathematical literacy development
3. Identify the goal of problem-solving and its development in the classroom
4. Compare and contrast mathematical literacy and language/reading/writing (literacy) development

Module B: Patterns
The paraeducator will:
1. Use concrete materials to aid pattern recognition and generalization
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3. Solve problems using geometric relationships and spatial reasoning (e.g., coordinate geometry, congruence, similarity)
4. Identify angle types
5. Use both standard and non-standard measurement for perimeter and area

Question 1: Common Responses

- Word problems (problem-solving)
- Numbers
- Rules
- Problems
- Memorization
- Skills
- Drill
- Homework
- Not fun
- Too hard

Question 2: Common Responses

- **Scientists (doctors, teachers, etc.)**

- **Smart people**

- **Parents (adults)**

- **People in jobs**
  (*stores, waiters/waitresses, etc.*)

- **No one – it is just for school**
  (*some young students believe this*)

Response Chart 3

Your Perceptions

-
-
-
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-

Arithmetic

Arithmetic is calculations involving predefined rules.

Mathematics is …

1. a study of patterns and relationships

2. a way of thinking

3. an art

4. a language

5. a tool

– Reys, Suydam, and Lindquist (1992)

Learning a Second Language

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Math as a Language

To learn the math language students/learners must:

1. Read it

2. Write it

3. Speak it

4. Do it

Math Journal

- ✔ **Reflections**
- ✔ **Problem-Solving**
- ✔ **Reference**

Problem-Solving: A Definition

Problem-solving means engaging in a task for which the solution method is not known in advance.

In order to find a solution, students must draw on their knowledge, and through this process, they often develop new mathematical understandings.

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Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking.

– NCTM (2000)

Problem-Solving

"I have five coins in my pocket,
some dimes, nickels, and pennies.
I pull three coins out. How much
money can I have in my hand?
What is the most money?
What is least amount of money?"

Skills and Concepts

- **Coin recognition**
- **Coin value recognition**
- **Adding money amounts**
- **Organizing data**
- **Decimal knowledge (depending on grade level)**
- **Greatest, least (magnitude)**

Problem-Solving with a Plan

Steps for Problem-Solving:

- **Understand the problem**
- **Devise a plan**
- **Carry out the plan**
- **Look Back**

Comparing and Contrasting Reading and Math Literacy

"What skills/expectations do you think a student has to be able to demonstrate order to have the appropriate language/reading/writing (literacy) skills needed in grades K-4?"

Reading	Language	Writing

Defining Literacy

Reading Literacy	Mathematical Literacy
Fundamentals	Fundamentals
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Comprehension	Ability to explain a problem or process
Prediction	Patterns and problem-solving
Communication	Communication

Module B
Instructor's Guide

Module B: Patterns

A. Lecture: Module Introduction
Use the **Module Goals** transparency and handout **(H1/T1)** to review the goals of the module.

Module B: Patterns
The paraeducator will:
1. Use concrete materials to aid pattern recognition and generalization
2. Relate basic patterns to algebraic concept development
3. Describe patterns and other relationships to interpret data using tables and graphs
4. Employ strategies of problem-solving to make predictions and determine the likelihood of an event

Goal 1: Use concrete materials to aid pattern recognition and generalization.

1.1 Activity: Patterns Galore

Paraeducators will recognize and extend several patterns and develop probing questions about the patterns.

Materials
- Pattern or attribute blocks or shapes (enough for entire class)
- Overhead projector
- Overhead pattern or attribute blocks or shapes (transparent colored shapes)
- Crayons (or other coloring tools)

Note: If patterns or attribute block are not available, this activity may be completed using any type of small object that can be "repeated" in a pattern and can be shared by the group.

1.1.1 Steps

- Introduce the activity to the entire class beginning with a brainstorming activity that will cause participants to think about all the patterns that surround them in their daily life. Use the **Patterns All Around Us** transparency **(T2)**. Direct class members to think about all of the places where patterns may be found in a classroom. Record responses on the transparency. Possible answers include:
  - ▲ Calendar
  - ▲ Carpet
  - ▲ Clothing
  - ▲ Music
  - ▲ Seating arrangements
  - ▲ Floor tiles

- Begin the lecture by placing transparent attribute shapes in a very basic pattern on the overhead. Use a simple pattern such as yellow hexagon, green triangle, yellow hexagon.
- Ask participants to share aloud possible patterns and predictions for the next shape.
  - ▲ Color
  - ▲ Shape
  - ▲ Size (if doing a pattern with attribute blocks)
- Explain that the ability to **recognize simple patterns and make predictions** involves the use of major mathematical skills.
- Using this simple pattern, ask the paraeducators to open their math journals to *Problem-Solving* and record questions that could be asked regarding this pattern such as,
  - ▲ What color is ***first*? (**an ordinal concept)
  - ▲ What will be the shape and color for the ***next*** piece? (a sequential concept)
  - ▲ ***How many*** shapes are in the current pattern? (a quantitative concept)
  - ▲ ***Adding one more*** shape will give how many shapes? (a quantitative concept)
- Explain that counting, shape, color, order, and number place are all important mathematical ideas/concepts and can be used to communicate mathematically.
- Divide participants into partners. Direct the paraeducators to create a more difficult pattern for a partner and repeat the same exercise as above: name the pattern, predict the next shape, explain how you knew what would come next, and develop questions that could be asked. Remind participants that this is an activity that occurs regularly in K-4 classrooms, especially in kindergarten and first grade, so it is highly likely that they will be assisting students with exactly this activity in the classroom.
- Provide time at the end of the activity for participants to record the information in the problem-solving portion of their journal.

1.2 Lecture: Patterns All Around Us

Patterns are usually the first mathematical concepts children experience even though their caregivers or the children themselves do not typically recognize or identify it as mathematics. Studies have shown that children recognize patterns as early as 3-1/2 years old. A child's ability to

- recognize,
- replicate,
- create,
- describe, and
- generalize patterns

is key to developing later mathematical skills (**Patterns All Around Us** handout **[H2]** and the transparency **Patterns [T3]**).

There are two main kinds of patterns: repeating patterns and growing patterns.

Repeating patterns: Elements in the patterns repeat according to some rule. Repeating patterns are an easy place to begin helping children with prediction skills.

Growing patterns: Elements in the patterns change (grow) according to some rule. These are also natural patterns, and are the introduction to algebraic concepts. The simplest growing pattern is counting.

Repeating patterns are exactly as one would expect from the name. Elements in the patterns repeat according to some rule, such as the simple pattern created in the activity completed earlier. Many classrooms have calendars with shapes as dates. The pattern could be cloud, umbrella, cloud, umbrella, etc. Repeating patterns are an easy place to begin to help children with prediction skills. Patterns can also be more complex such as having two elements repeat: triangle, triangle, square, triangle, triangle, square. In this pattern, students could see individual elements as the repeating pattern or could see the shapes in terms of groups of three.

Growing patterns contain elements that change (grow) according to some rule. These are also natural patterns, and are the introduction to algebraic concepts. The simplest growing pattern is counting, such as dates on the calendar. For students who have difficulty counting, it is often helpful to treat counting as a pattern. For example, when students reach 10, the first number is 1 and the last number is increasing from 0 to 9. When we reach 20, a similar pattern exists. Growing patterns, even in kindergarten, are the introduction to algebraic concepts. This will be addressed in the next few goals.

1.3 Activity: Barnyard Counting (Optional)

Paraeducators will recognize counting patterns and use games to emphasize counting skills.

1.3.1 Steps

- Direct participants to form a large circle around the outside of the room.
- Explain that counting games are very important. They can help students focus on patterns and increase students' ability to communicate mathematical ideas. Point out that the following activity is for young students and that there will be a second activity that may be used with older students.
- Ask each participant to choose a barnyard animal sound such as mooing or oinking.
- Explain that when young students begin working with patterns, they typically start with simple patterns such as odd or even, or counting by 2's and by 5's.
- Begin the activity with the pattern for even numbers. For this activity define counting even numbers as counting by 2's.
- Before starting, make sure that everyone agrees what the first even number is.

- The first person counts aloud, saying “1.”
- On “2,” since it is even, the second person makes a barnyard sound.
- The third person says “3.” Then the fourth person makes another barnyard sound, etc. Continue around the room or circle several times, watching the pattern repeat and grow.
- Try the same with different patterns, counting by 3’s or 5’s, for example.
- Explain that the next part of the activity is an example of how to use the activity with older students. This time ask participants to pick a sound that they think is fun and that would give a more “rap” or musical feeling to the activity. Possible sounds include tongue clicks, buzzing, finger snaps, or any other sound that a participant would like to attempt.

At the end of the activity direct participants to open their journals to *Reflections* and take a few minutes to record their thoughts about the activity and some ideas about how and when they could use their learning to assist students. If time permits, ask them to share some of their ideas with other class members.

Goal 2: Relate basic patterns to algebraic concept development.

2.1 Activity: “Rules Rule”

Paraeducators will gain knowledge of how to create rules to sort and classify objects.

2.1.1 Steps

- Ask each paraeducator to take two objects out of a purse, pocket, or book bag and set them aside for the activity.
- Divide the class into small groups of 4 or 5. Each small group will need a table or work surface to complete the activity.
- Ask participants to place the objects that they removed from their purses, pockets, or book bags on the table.
- Ask each group to sort their combined objects into 2 or 3 groups/classifications.
- Direct the groups to designate a note taker to represent them. Members at each table should determine the reason for their classification and give it a rule. The note taker should record the rule.
- A rule should cover the entire classification such as red and not red.
- Direct the groups to share their classifications with the rest of the class.
- Next, ask each group to reclassify their objects according to a new set of rules.
- Explain to class members that they have been engaging in an activity that gave them an opportunity to classify by a variety of rules. Possible rules include:
  - ▲ Attributes (color, size, shape)
  - ▲ Use
  - ▲ Number

- Use the **Algebra** handout and transparency **(H3/T4)** to explain that:

  The ability to classify and sort objects by rules is the beginning of algebraic concepts.

- Repeat the activity using attribute blocks. The note taker should record the rules as the groups sort and classify by shape names, sizes, colors, and other attributes that they notice as they work with the blocks.

2.2 Lecture: Algebra in Early-Elementary Grades

When the term "algebra" is used, people think of letters, equations, and high school mathematics classes. While this is true, algebra is actually a concept that is introduced when students solve their first addition problem.

Review the **Algebra** handout and transparence **(H3/T4).**

Algebra

Algebra is a method by which to describe and predict the behavior of a set of data. *Data* can be anything: blocks, toys, weather, numbers, etc. In other words, our spoken language can be represented as written words that can be read and have meaning.

Spoken words consist of representative letters that convey meaning to the reader when they have been written down. The same happens in math. We represent data such as objects, actions, numbers, etc., in written form. We use simple or complex mathematical "words" or "sentences" to represent the data. For example, "Two oranges put in a basket with two more oranges results in four oranges being in a basket," or 2 + 2 = 4. Sometimes the "2" represents oranges, sometimes it represents DNA strands; it can be as simple or as complicated as the context surrounding it.

Young children should be encouraged to explore mathematical relationships using words, symbols, pictures, and objects. Algebraic thinking starts with patterns. Patterns can be expressed as mathematical symbols from the written or oral communication of the pattern. Students need practice describing what they see happening. They must generalize what they see in order to help them predict. An important aspect of algebra is creating a rule to represent the data. Rules, such as the ones used in the sorting and classifying activity, can be expanded into numeric concepts.

2.3 Activity: Step Right Up!

Paraeducators will define a variable and use basic algebra skills to predict values.

Materials

- Overhead projector
- Small paper cups
- Chips, discs, beans (any small object for counting)
- Pattern or attribute blocks for class members
- Clear pattern or attribute blocks and counters for use with overhead projector

2.3.1 Steps

Divide the class into pairs and instruct them using the following steps:

- Give each pair a paper cup and 10 counters and one pattern block of any shape.
- Explain that algebra can be useful in teaching basic addition or subtraction concepts.
- Explain that each pair has 10 counters.
- Without telling the group, place 7 counters under a cup on the overhead and leave 3 out that they can see.
- Ask the group to predict how many counters are under the cup. Most will easily get the answer of 7.
- Explain that this is a very difficult task for young children. Some children are not developmentally ready to understand what happened and how they could possibly know how many counters were under the cup.
- Ask the group what the problem is and what information is given.
  - ▲ We need to know how many counters are under the cup.
  - ▲ We know there are 3 on the outside and there were 10 total.
- Help the group write an algebraic model using the ***missing-addend model.***
  - ▲ Ask the group how many they can see. (3)
  - ▲ Ask the group what operation will be necessary; addition. (+)
  - ▲ Use a shape (triangle) and place that on the overhead to represent what we don't know.
  - ▲ Ask how many they had total. (10)
- The **algebraic equation** they wrote is *3 + triangle = 10.*
- Ask class members what methods could be used to figure out how many counters are under the cup:
  - ▲ Start at 3 and count up
  - ▲ Count down from 10
  - ▲ Act out the problem with other counters
- In this algebraic equation using the ***missing-addend model,*** triangle = 7.
- Provide another example (5+5=10) using the same triangle.
- This time, the equation is *5 + triangle = 10* (triangle = 5).
- Ask how the triangle can equal 5 and 7 and several other numbers?
  - ▲ Triangle is a *variable*
  - ▲ Use the **Variable** handout and transparency **(H4/T5)** to define the term.

    A *variable* is a symbol or object that can change value depending on the problem in which it appears. It simply holds a place until it has a value, then it changes with the next problem.

Variable
- A symbol or object that can change value depending on the problem in which it appears.
- Simply holds a place until is has a value, then it changes with the next problem
- Its value can vary, or is ‘variable’

- Ask what type of patterns existed in this activity
  - ▲ Was a **repeated pattern**; it equaled 10 each time because the total number of counters did not change
  - ▲ Worked with addition
- Ask what observations could be made throughout this activity
  - ▲ Had a constant answer of 10
  - ▲ Used addition concepts each time (it does work with other operations as well)
  - ▲ Each equation had a variable (didn’t matter what shape was used to hold the place)
- This example shows that students are doing algebra from the first days of school when they fill in missing numbers in boxes or in response to question marks.
- Direct working pairs to practice writing algebraic equations using their shapes.
- Direct participants to record new terms and definitions in their math journal *Reference* section. Make sure to provide time throughout this Academy for reflections before moving to new material.

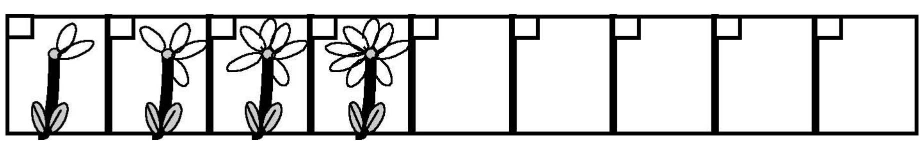
2.4 Activity: Grow a Pattern

Paraeducators will expand rules and patterns into algebraic expressions.

2.4.1 Steps

- Use the **Grow a Pattern** transparency **(T6)** to introduce this activity.

Grow a Pattern



- Ask class members: “What is changing with each picture?”
  Answer:
  - ▲ The number of petals is increasing.
- Write the number of petals in each box in the upper left-hand corner of the frame.
- Ask: “What is staying the same in each frame?”
  Answer:
  - ▲ All have stems.
  - ▲ All have two leaves.
  - ▲ All have a center.

- Use the **Grow a Pattern** handout **(H5)** and ask attendees to proceed with the instructions. See below for detailed discussion points.

  1. In the upper left-hand box of each frame, write the number of petals you see.
  2. Draw the next flower. How many petals can you predict it will have? Why might drawing further flowers be difficult? Is it necessary?
  3. In the remaining frames, record your prediction for the number of petals.

  Use your math journal to record ideas and thinking for the rest of the activity.

  4. Read questions 5 and 6 on the handout. We need a way to organize our data to help with our prediction. With your partner, discuss how to organize your data. Record your method and the organized data. Be ready to share with the group.
  5. What flower will have 20 petals?
- Explain that we need some way to organize the data for the petals to be able to see any patterns that will help us predict.
- (For question #4) Ask attendees to share with the class the types of charts and tables and other ideas they came up with.
  - ▲ Guide them towards giving each frame a number (*pattern numbers*). Help them see that at this point it is good idea to number each box; the purpose for the numbering is that it is a way to begin to record data. It is a more accurate way to record the data.
  - ▲ Explain that pattern numbers help when discussing concepts
    - o it is very difficult to reference a picture without giving it a name or a number
    - o by numbering the patterns, we can refer to them and their attributes (e.g., number of petals or leaves)
    - o it assists in seeing patterns that are otherwise hidden
  - ▲ If needed, use the overhead or chalkboard to create simple examples of tables or charts that have not already been discussed such as the one below.

Pattern Number	Petals
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18

- Review what has probably already been discussed about the patterns that have been identified.
  Answers and discussion suggestions:
  - ▲ The number of petals is increasing by 2 each time.
  - ▲ All flowers have even numbers of petals. If somebody has not already mentioned this pattern, ask the question, “Which flower would have 27 petals?” or “33 petals?”
    - o Allow the class to think about it and respond that “the pattern is increasing by 2, so all flowers will have an even number of petals;” “no flowers could have 27 or 33 petals.”
    - o Some members might conclude that “some flowers could have **at least** 27 or 33 petals but none would have that total amount,” because “an odd number does not fit the pattern.”
  - ▲ Twice as much as the pattern number.
    Instructor note: This response will probably not come up at this point. Do not force the response as students will discover this pattern throughout the activity.
- Introduce the idea that to get to the 10th flower or something higher, we would have to draw a lot of petals; we need something to help us predict.
- Ask participants to again look at the pattern number compared to the number of petals and think about a way to reorganize their tables (refer to the example of a table that you put on the overhead or chalkboard).
- If it does not arise naturally, guide participants to see that “the flower (pattern) number, if doubled, will give the number of petals” – write this out in words.
  - ▲ *If we double the flower number, we can predict the number of petals on any flower.*
  - ▲ To guide this thinking, use questions like: “What is the relationship between the pattern number and the number of petals on each flower?”
  - ▲ Direct the attendees to the table, pointing at the number in each cell, identifying the pattern number, and then pointing to the number in the box, or of petals on each flower.
- Explain that they will now begin to write an *equation*. The equation is a way to keep track of the flower (pattern) numbers. Ask class members to refer to the activity that they did with the cup and chips. Ask how they represented the number of chips that were under the cup; what variable did they use? Many of them used a triangle as a variable. Within the flower petal equation, we need a variable. We could use a variable such as “the flower itself.” Draw the flower.
- We would like to use our equation to express that we know that if we “*double the flower number, we can predict the number of petals on any flower.*” We want to indicate or show that we are “doubling” the flower number. Doubling is also expressed as “2 times.” We can write:

2 x (flower) = # petals

- Keep in mind that in traditional algebra we use letters to represent unknown quantities (*variables*). In this case we can use a letter such as *f* for the flower (pattern) number. We express doubling the flower number as: **2 x *f* or 2*f*.**
- This written numeric expression helps us to predict the number of petals. This connection is very important because it says:
  - ▲ The pattern number times 2 gives the number of petals on the flower.
- Ask attendees to answer the last two questions:
  - ▲ How many petals does the 20th flower have?
    Answer: 40 petals
  - ▲ What flower will have 20 petals?
    Answer: The 10th flower
- Direct the class members to use the *Reflections* section of their math journal to record their thoughts and ideas about what they just learned about:
  - ▲ Patterns
  - ▲ Variables
  - ▲ Prediction
  - ▲ Equations
  - ▲ Any other personal ideas or thoughts, especially "ah-ha" moments.

Goal 3: Describe patterns and other relationships to interpret data using tables and graphs.

3.1 Activity: Troublesome Triangles

Paraeducators will use tables to track data and look for patterns.
Paraeducators will interpret graphs of data to predict outcomes.

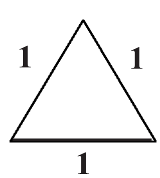
3.1.1 Steps

- Use the **Troublesome Triangles** handout and transparency **(H6/T7)** for this activity.

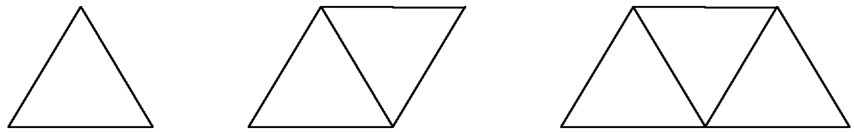
  ****Note to Instructor:*** You can also use the overhead projector with clear shapes and a transparency to write on while explaining this activity

- Referring to the handout, transparency and/or the overhead projector, use triangles to create a growing pattern as shown below. Explain that each edge is a portion of the perimeter and equals 1. Lines through the middle are not part of the perimeter and are not edges. In other words, this triangle has a perimeter of 3.

  Example:



Troublesome Triangles



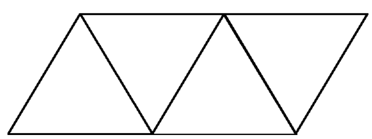
- Create a table to organize the data.

Pattern #	# of Triangles	Perimeter
1	1	3
2	2	4
3	3	5

- Divide the class into small working groups, about 3 per group. Direct the groups to follow the directions and complete their handouts:
  1. Draw the next figure in the pattern.
  2. Predict how many triangles will be in the 5th figure before drawing it. What pattern have you already established about the pattern numbers?
  3. What is the perimeter of the 3rd figure? 4th figure?
     Remember:
     - *Perimeter* means the length of the outside edges.
     - Figure 2 has a perimeter of 4 because you do not count the middle line.
  4. What is the perimeter for the 5th figure?
  5. Describe a pattern for the number of triangles.
  6. Describe a pattern for the perimeter.
  7. Write an algebraic expression for the number of triangles in future figures.
  8. Write an algebraic expression for the perimeter of future figures.
  9. How many triangles are in the 20th figure? (do not draw them)
  10. What is the perimeter of the 20th figure? (do not draw them)

Answers:

1.



2. 5, each figure increases by 1 triangle each time
3. 5; 6.
4. 7.
5. It is the same as #2; it is the same as the pattern number and increases by 1 each time
6. It is 2 more than the pattern number; increases by 1 each time, starting at 3.
7. “S” is an algebraic expression for the **number** of triangles in future figures. “S” or “sides” is the same as the pattern number. Any variable may be used.
8. s + 2 is an algebraic expression for the **perimeter** of future figures. s + 2 always depends on the pattern number.
9. 20 triangles.
10. Perimeter is 22.

3.2 Lecture: Tables and Graphs

Remind class members to use their math journals to record definitions. Use the transparency and handout **Tables and Graphs (T8/H7)** to support this discussion. The use of tables has already been introduced in the previous activity. Tables are used to organize data. Organized data are important in pattern recognition and developing algebraic expressions. Emphasize that tables and graphs are used extensively in early mathematics to help students learn to organize and understand information. The more familiar the paraeducators are with tables and graphs, the better able they will be to assist students in math classes and other academic activities that suggest a need for organization of data, such as science activities, art, and literacy opportunities.

Tables and Graphs

Tables are used to organize data. Organized data are important in pattern recognition and developing algebraic expression. Using a table is a way to record *frequency (number)* to show how often an item, a number, or a range of numbers occurs. Tables help organize, summarize, and simplify.

Graph*s* are another mathematical tool for organizing data, and are easily integrated throughout the elementary curriculum. Graphs can be used to show the number of students present daily, temperature changes throughout the day, or the *frequency (number)* of an attribute such as the number of students who have dogs. Graphs can be used for a one-time study or a study that looks for trends. Graphs are simply a way to picture data.

3.3 Activity: The Use of Tables

Paraeducators will review the use of tables that students in K-4 classrooms might be exposed to during their academic day.

Students in K-4 classrooms are exposed to the use of tables throughout their classroom day. Again, tables help *organize, summarize, and simplify* the data that we interact with every day.

3.3.1 Steps

- Divide class into groups of 4-5.
- Distribute the **The Use of Tables** handout **(H8).**
- Review the tables used as examples in the handout. Remind participants that these examples are but a few of the types of tables that students frequently see.
- Direct groups to respond to the question at the bottom of the handout.

The Use of Tables
Math story problem
List of daily temperatures
Table of contents

- As a group discuss and record your responses to the following question:

  "How do you think these sample tables would help a student organize, summarize and simplify data?"
- Review group responses with entire class.

3.4 Lecture: Types of Graphs

Use the **Types of Graphs** handout and transparency **(H9/T9).** Explain that there are a variety of graphs and that participants should be familiar with them. Review the graphs in the handout. Some of the most commonly used include the following:
- object graphs
- picture graphs
- line plot
- bar graphs
- line graphs
- circle graphs (pie charts)

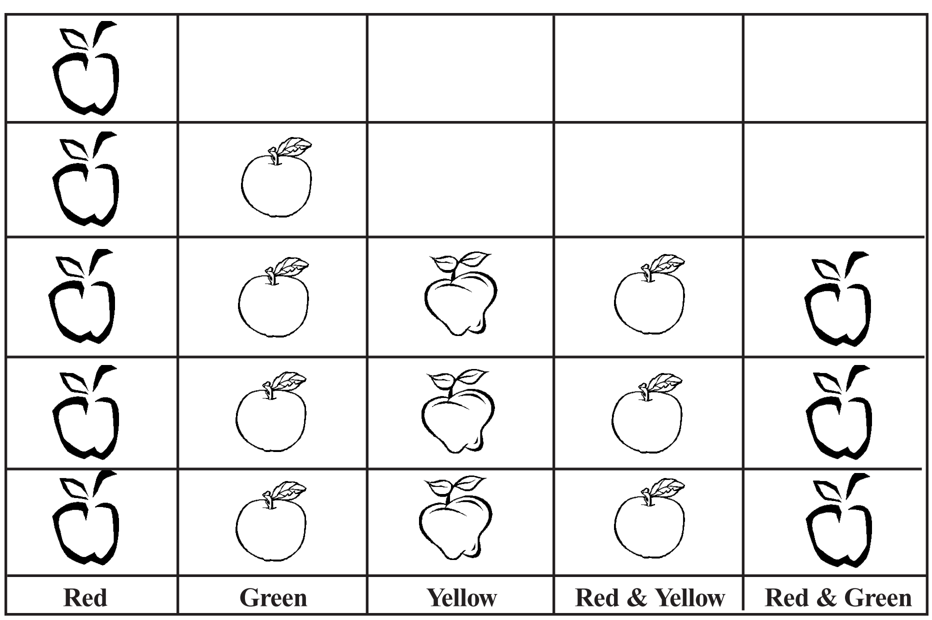
Types of Graphs

Object Graph: This type of graph uses the actual object to display frequency (number).

Example: The teacher asks every student in the class to bring an apple to school. The teacher creates a blank grid on a large sheet of paper on the floor. With the help of the students, the grid is labeled to represent the types of apples they brought. The labeling could be by color, type of apple, taste, etc.

The apples in this activity would be real apples, not picture representations.

Types of Apples Brought by Ms. Sees' Class



Ask participants to discuss the pros and cons of this type of graph.

- The graph is easy for children to create and interpret.
- The graph is limited by set and object size.

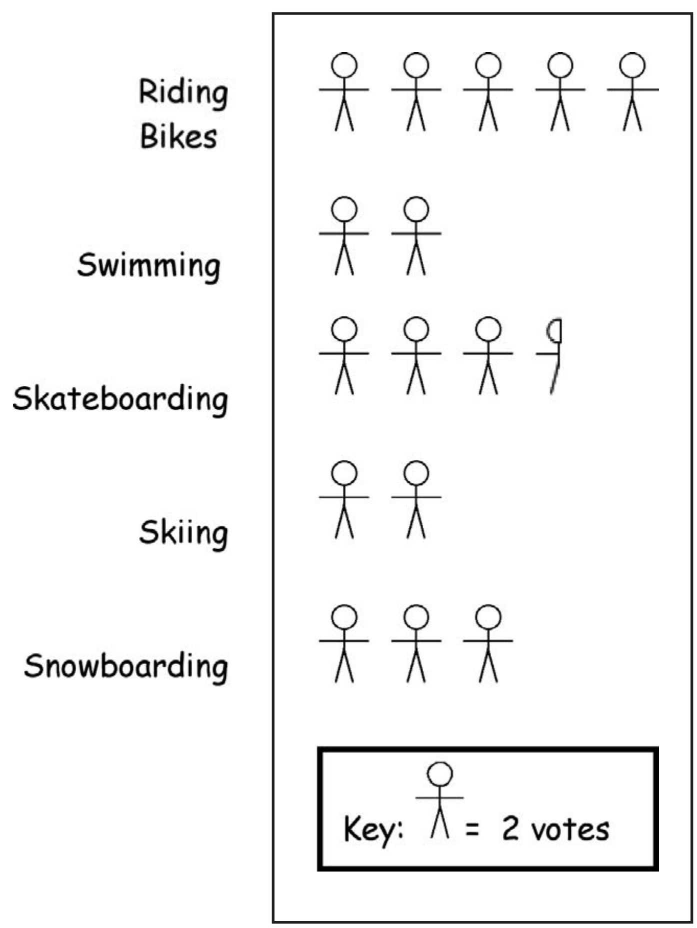
***Pictographs or Picture Graphs*:** This type of graph uses pictures or symbols to represent and compare data.

Example: A teacher asks her students to poll each other regarding their favorite recreational activity.

The results may be displayed using a picto- or picture graph. A key is vital to this graph to show what each symbol or picture stands for.

Sample Graph

Favorite Types of Winter Recreation



Discuss the data for skateboarding. Note the half symbol. Discuss that is would stand for 1 vote rather than 2.

Ask participants to discuss the pros and cons of this type of graph.

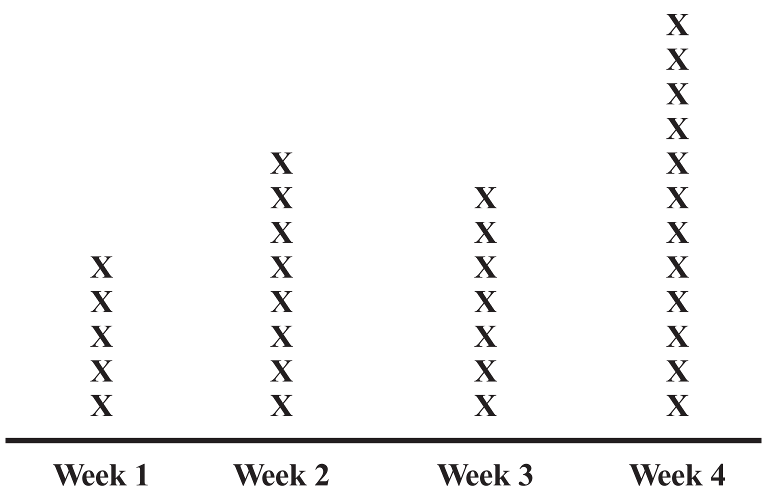
- Students generally read these graphs well if they pay attention to the key.
- Drawings can be time consuming if student-created but the data can be easily read.
- The graphs handle large data sets well by adjusting the key.

Line Plots: Line plots are used to show frequency and can show comparison of data.

Example: A reading teacher asks her small group to keep track of how many books they finish per week for one month. The teacher totals the number of books read each week. The teacher enters the data into a simple table.

Weeks	Books
Week 1	5
Week 2	8
Week 3	7
Week 4	12

The data from the table are then plotted on a line plot to graphically show the distribution of the data. This implies that the data will have a range from highest point to lowest point.



Ask participants to draw conclusions from the plot:

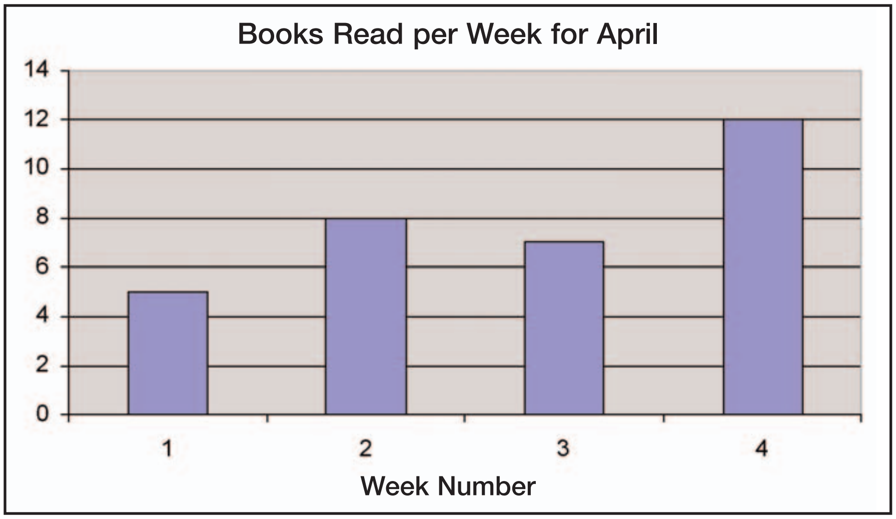
- The range is 7 books (highest value minus lowest value).
- A total of 12 books was the highest number read, which occurred in Week 4.
- A total of 5 books was the lowest number read, which occurred in the first week.
- Students seemed to get more motivated or got better at reading (many reasons are possible here) as time passed.

Ask participants to discuss the pros and cons of this type of graph.

- Line plots are easy to create and modify with new data.
- They are easy to interpret.
- Can be time consuming for large amounts of data.

Bar Graphs: In a simple bar graph the length of the bars represents numbers (frequency).

Example: The data from the data table on the previous table, representing the number of books read for one month, may be used as an example of the use of the bar graph.



Ask participants to discuss the pros and cons of this type of graph.

- A bar graph is often used because it is easy to show most or least.
- It is easy to create.
- Can lose track of individual data points, so it is often necessary to estimate the height of bars to read the data.

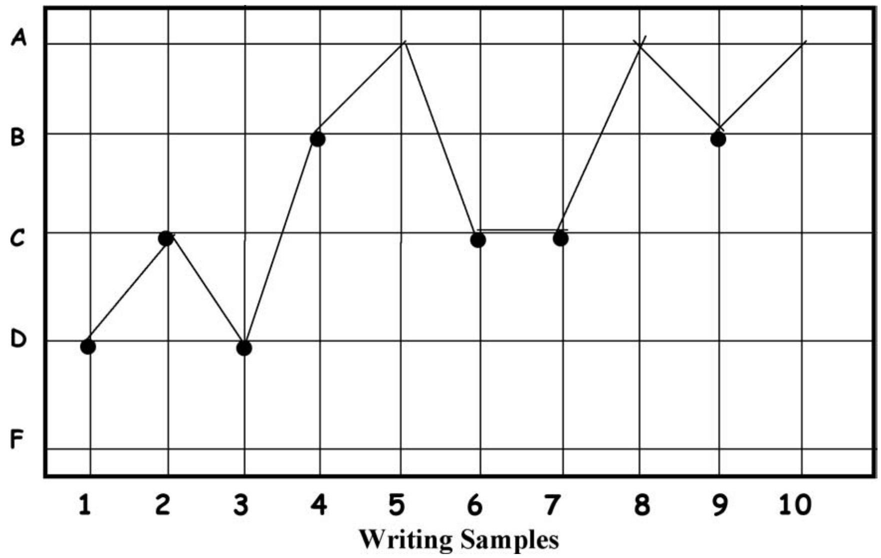
Line Graphs: Most graphs include some sort of measurement that shows time. When you look at the line in a line graph, you can tell whether something has increased, decreased, or stayed the same over time. Line graphs show trends in the data.

Example: A teacher would like to demonstrate to her 6th-grade student that he has made significant change in his writing skills in a more graphic way than just showing his grades. She enters the data from her grade book in a line graph for 10 writing samples.

Grade Book
Timothy's grades

Writing Samples:	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Grades:	**D**	**C**	**D**	**B**	**A**	**C**	**C**	**A**	**B**	**A**

Timonthy's grades plotted on a line graph



Ask participants to discuss the pros and cons of this type of graph.

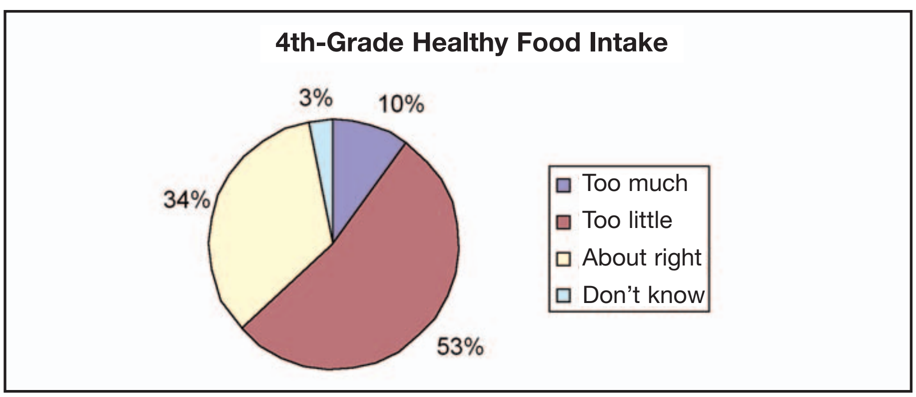
- It is easy to see trends.
- Could be overly confusing if several students' data were shown on a single graph.
- Can be easily skewed by the graph scale.

***Circle Graphs**:* These are a helpful way to organize data. Circle graphs are sometimes call pie charts.

Example: A 4th-grade teacher asked her class to poll all 4th-grade students in the school regarding healthy food. Students responded, and the data were put in a table that indicated intake of healthy food across an average day.

Too much	Too little	About right	Don't know
10	53	34	3

Here is a circle graph of these data:



Ask participants to discuss the pros and cons of this type of graph.

- Must have grade-level appropriate expectations for accuracy of percentages.
- It is difficult to divide a circle.
- Can be easily interpreted once created (most, least, etc.).

Keep in mind: Any type of tool that is used to represent data lends itself to a variety of interpretations. In the public media data are used constantly to make a point: a political poll, data for a product to be sold, or a comparison of prices. Paraeducators should be able to help students realize that charts and graphs can be skewed for any particular purpose. Students may need reminders that they are always to look at the keys and titles to determine the accuracy of a graph. Students should have experience with choosing appropriate tools to represent their own data.

Creation of graphs and tables as well as the interpretation of the data patterns are key skills for mathematical development and communication. Paraeducators should feel confident in their ability to help students to their best understanding and use of skills related to charts and graphs.

3.5 Activity: Graph It

Paraeducators will use their pattern skills to create and interpret various graph types.

Materials

- Sticky pad notes
- Centimeter or large squared graph paper (**Graph Paper** handout **[H10])**
- Plain paper that can be used for drawing
- Markers
- Crayons (or other coloring tools)

3.5.1 Steps

- Direct participants in small groups to review the **Types of Graphs** handout **(H9).**
- As a class, complete one data set to be graphed by the entire to class to link one graph type to another.
- Suggest ideas such as "numbers of pets," "favorite food," or "number of people in the family."
- Collect class data once the topic has been chosen. Post the data.
- Provide each group with the materials listed above to complete their activities.
- The small groups are to graph the class data using each of the graph types from the handout. If the line graph (showing trends) is not appropriate, it may be excluded from this exercise.
- Sticky notes can be used to represent the objects (draw pictures) for the object graph.
- After participants have completed the activity, ask them to reassemble as a large class and review their data/graphing samples with the entire class.

- Make sure to check for graph elements such as titles, axis labels, and appropriate format. Typically, the idea being measured is listed along the horizontal axis (x-axis) and the frequency on the vertical axis (y-axis).
- Direct class members to use the *Reflections* portion of their math journals to record personal ideas, thoughts and "ah-ha" moments for personal graphing as well as graphing experiences with students.

Goal 4: Employ problem-solving strategies to make predictions and determine the likelihood of an event happening.

4.1 Activity: Flipping Out

Paraeducators will use their pattern abilities to make valid predictions.

Materials

- 1 coin for each small group
- Transparency and handout **Probability (H11/T10)**

4.1.1 Steps

- Divide the class into small groups of 3-4.
- Direct each group to flip their coin 10 times and record the data using the method of their choice.
- Ask each group to discuss their results and post the results on the chalkboard or on the overhead.
- Ask class members to predict what the next coin toss will be: head or tails.
- After class members have made their predictions, discuss the following points with them using the **Probability** handout and transparency **(H11/T10).**
  - ▲ The answer is simply a guess, as head and tails are *equally likely;* that is, there is equal *probability (chance)* for either prediction.
- Direct each group to flip their coins one more time and share their results.
  - ▲ The results should be about equal because heads and tails are equally probable.
- Discuss why this is the case for a coin.
  - ▲ How many options are possible? (2)
  - ▲ Is one side more likely than the other? No – *both are equally likely.*
  - ▲ This coin is considered a *fair* coin as the options are equally likely. That is why we generally are satisfied with the results of a coin toss when we are choosing which team kicks off or which receives the ball, or who gets a particular privilege. Using a coin toss to provide a choice does not usually end in a cry of "unfair" because there is an equal chance for each side. Each side sees the results as *fair.*
- Propose the concept of *likelihood*. Explain that the term used for the concept of likelihood in mathematics is "probability." In other words, how "probable" it is that something will happen. Explain that the concept of

probability or likelihood can be taught to young students using things that happen in their everyday life. One example would be weather predictions. For example:
- ▲ 70% chance for rain – *likely*. There are 7 out of 10 chances, or a 70% probability that it will rain.
- ▲ 10% chance for rain – *not likely*. There is a 1 out of 10 chance, or a 10% probability that it will rain.

- Propose other questions to look at important terms:
  - ▲ What is the likelihood of flipping a star on a coin? None – *impossible*
  - ▲ What is the likelihood of flipping a heads or tails? *Certain*

4.2 Activity: Practicing Probability

Paraeducators will use a spin chart to better understand the concept of probability.

Sometimes it is difficult for students to understand terms and concepts when we have included terms like "percentage" or "likelihood." A hands-on activity is a good way to help students internalize the concepts and the corresponding language.

Materials
- 1 paper clip per group
- **Spinners** handout **(H12)**
- Crayons (other coloring tools)

4.2.1 Steps

- Divide the class into pairs.
- Hand out a copy of the **Spinners** handout to each pair. (Give each class member a copy of the handout for their notebooks and then give each pair a separate copy to use for the activity.)
  ***Note to Instructor:* The following activity material that is within parenthesis is for instructor information only and should not be given to the participants.**
- Direct each pair to use their paper handouts and a paper clip to create a spinner that:
  - ▲ Uses the numbers 1-4, where the number 1 is the most likely ***(should divide circle where the 1 area has largest space).*** Working pairs should record their responses as directed in the handout.
  - ▲ That is *fair* for the numbers 1-4 ***(4 equal regions)***. Working pairs should record their responses as directed in the handout.
  - ▲ Makes 3 an almost impossible result for the game (***3 has a very tiny area that is nearly impossible to land on).*** Working pairs should record their responses as directed in the handout.
- After completing all three spinner activities, divide the class into 2-3 larger groups and ask them to discuss their results with each other.
- Direct the groups to write about the activity in their journals under *Reflections*.

- Ask groups to respond to the following question (remind them to take notes in their math journals):
  How could you use this activity to further support students in following areas in math?
  ▲ Fractions
  ▲ Percent
  ▲ Decimals
  ▲ Comparison
  ▲ Odds
  ▲ Prediction
  ▲ Science Concepts

Assignment Part 1 (of 2 parts)

****Note to Instructor:*** You will need to decide how much time to give the class to complete their assignments so that you have time to grade, record grades, and turn in materials from this course in a timely manner. If paraeducators are taking the course for credit, there will be a time limit based upon the grading period at the attending institution. You also need to decide how you would like attendees to turn in their assignments. Options include mailing or whatever arrangements work for you and your class. **You are strongly encouraged to be firm about a completion date and may need to make some effort to follow up on attendees and their progress**. **Refer to the Grading Rubric handout (GR) for details on grading.**

Assignment: Seeing Graphs
Hand out **Assignment Part 1, Seeing Graphs** (**H13).** Read the instructions and answer the questions regarding completion of the assignment. Provide the class with a date for completion and your process for handing the assignment in.

The following assignment is worth **125 total points**.

The focus of the assignment is the use of tables and graphs to represent data. Participants will be finding real data examples and analyzing them according to class materials.

There are three steps to this assignment.

Step 1: (10 points)
Participants will find an example of a graph in the newspaper, a magazine, the Internet, or other source. The participant must make a copy of the source to attach to the assignment. Accurate completion of this step of the assignment is worth a maximum of 10 points.

Step 2: (80 points)
Participants are to answer the questions on the assignment about their graph choice. This includes naming the graph type, data graphed, and other conclusions. Accurate completion of this step of the assignment is worth a maximum of 80 points.

Step 3: (35 points)
The last part of the assignments asks participants to write five questions about their graph that could be used with students. An answer key must be provided. Accurate completion of this step is worth a maximum of 35 points.

Assignment 1 Handout: Seeing Graphs

Name:

Date:

Step 1: (10 points)
Find an example of a graph in the newspaper, a magazine, the Internet, or other resource. Make a copy of that source to attach to the assignment.

Step 2: (80 points)
Answer the following questions about your graph.

1. Name the type of graph from the **Graph Types** handout that matches your example.
2. Give the title of your graph.
3. What kind of information is your graph measuring?
4. List the key (legend) or list the labels for the concepts on your graph being measured (vertical and horizontal direction).
5. Is it easy to read and understand? If not, provide examples of what you would do to make it easier to read.

Step 3: (35points)

Write 5 questions about your graph that could be used answered by students. An answer key must be provided for your questions.

Module B
Handouts

Module Goals
Module B: Patterns

The paraeducator will:

1. Use concrete materials to aid pattern recognition and generalization
2. Relate basic patterns to algebraic concept development
3. Describe patterns and other relationships to interpret data using tables and graphs
4. Employ strategies of problem-solving to make predictions and determine the likelihood of an event

Patterns All Around Us

Patterns are usually the first mathematical concepts children experience. Children typically recognize patterns as early as 3-1/2 years old.

A child's ability to:

- recognize,
- replicate,
- create,
- describe, and
- generalize patterns

is the key concept to developing later mathematical skills.

Repeating patterns: Elements in the patterns repeat according to some rule. Repeating patterns are an easy place to begin helping children with prediction skills.

Growing patterns: Elements in the patterns change (grow) according to some rule. These are also natural patterns, and are the introduction to algebraic concepts. The simplest growing pattern is counting.

Algebra

The ability to classify and sort objects by rules is the beginning of algebraic concepts.

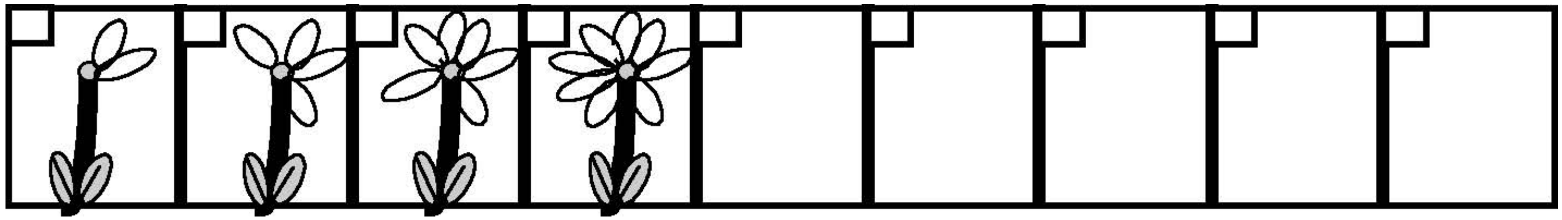
***Algebra* is a method by which to describe and predict the behavior of a set of data.**

***Data* can be anything: blocks, toys, weather, numbers, etc.**

Algebraic thinking starts with patterns.

Variable

- Is a symbol or object that can change value depending on the problem where it appears.
- Simply holds a place until is has a value, then it changes with the next problem.
- Its value can vary or is “variable.”



Grow a Pattern

1. Write the number of petals in the box in the upper left-hand corner of frames 2-4.

2. Draw the next flower. How many petals can you predict it will have? Can the number of petals be represented with a numeral or do you have to draw the flower each time to be accurate?

3. In frames 6-9, write the numbers of petals that you predict will be on each flower.

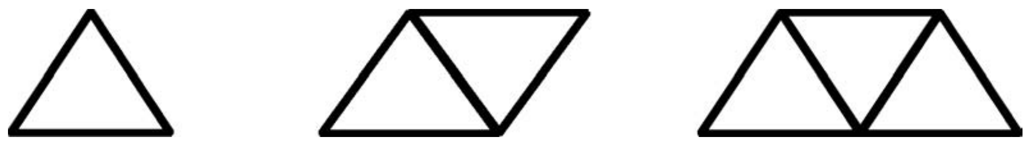
4. What number of flower will have 20 petals?

5. Use your math journal to record your ideas and thinking for the rest of the activity.

6. Work with a partner.
   - Think about ways to organize the data that you could collect about the flower petals.
   - Discuss the patterns that you think would help you organize your data.
   - Can you think of a table or chart that would be helpful for organizing your data about the flowers?
   - Draw a table or chart and put some data in it.
   - Be prepared to share your ideas with the class.

Troublesome Triangles

1. Working with a small group: Draw the next figure in the pattern below.



2. Predict how many triangles will be in the 5th figure before drawing it. What pattern have you already established about the pattern numbers?

3. What is the perimeter of the 3rd figure? 4th figure?
   Remember:
   - Perimeter means the length of the outside edges.
   - Figure 2 has a perimeter of 4 because you do not count the middle line.

4. What is the perimeter for the 5th figure?

5. Describe a pattern for the number of triangles.

6. Describe a pattern for the perimeter.

7. How many triangles are in the 20th figure? (do not draw them)

8. What is the perimeter of the 20th figure? (do not draw them)

9. Write an algebraic expression for the number of triangles in future figures.

10. Write an algebraic expression for the perimeter of future figures.

Tables and Graphs

Tables

Tables are used to organize data. Organized data are important in pattern recognition and developing algebraic expressions. Using a table is a way to record frequency (number) to show how often an item, a number, or a range of numbers occurs. Tables help organize, summarize, and simplify.

Graphs

Graphs are another mathematical tool for organizing data. Graphs are easily integrated throughout the elementary curriculum. For example, graphs can be used to show the number of students present daily, temperature changes throughout the day, or the frequency (number) of an attribute such as the number of students who have dogs. Graphs can be used for a one-time study or a study that looks for trends. Graphs are simply a way to help us picture data.

The Use of Tables

The following are examples of tables that are being used to help organize, summarize, and simplify data (information).

Example 1: A table containing a word problem from a math workbook.

Keisha bought a new CD. It cost her $14.68 plus tax. The total that she paid was $15.78. She paid with a 20-dollar bill. How much change should she receive?

1. Begin with the amount Keisha owes for her CD.	**$15.78**				
2. Count up from $15.78. Use coins to get to the next dollar.	**$15.78**	$15.79 1¢	$15.80 1¢	$15.90 10¢	**$16.00** 10¢
3. Count up with bills to reach $20.00.	**$15.78**	$17.00 $1	$18.00 $1	$19.00 $1	$20.00 $1

Example 2: Times and corresponding temperatures.

Evening Temperatures
(8:00 P. M.)

Time	Temperature
March 8	46°
March 9	41°
March 10	38°
March 11	29°
March 12	31°

The Use of Tables (continued)

Example 3: Sample table of contents.

The Life and Times of Paulette Paraeducator

Table of Contents .... *iii*

Chapter One: A Tough Job But Somebody's Gotta Do It................................. *1*
I Just Love Those Kids *5*
My Favorite Things.. *9*
What to Do When All Else Fails *14*

Chapter Two: OH NO, I've Been Assigned to Cafeteria Duty *18*
Effective Use of the Light Switch *21*
How to Eat and Run Without an Upset Stomach *24*

- As a group discuss and record your responses to the following question:

  "How do you think these sample tables would help a student organize, summarize and simplify data?"

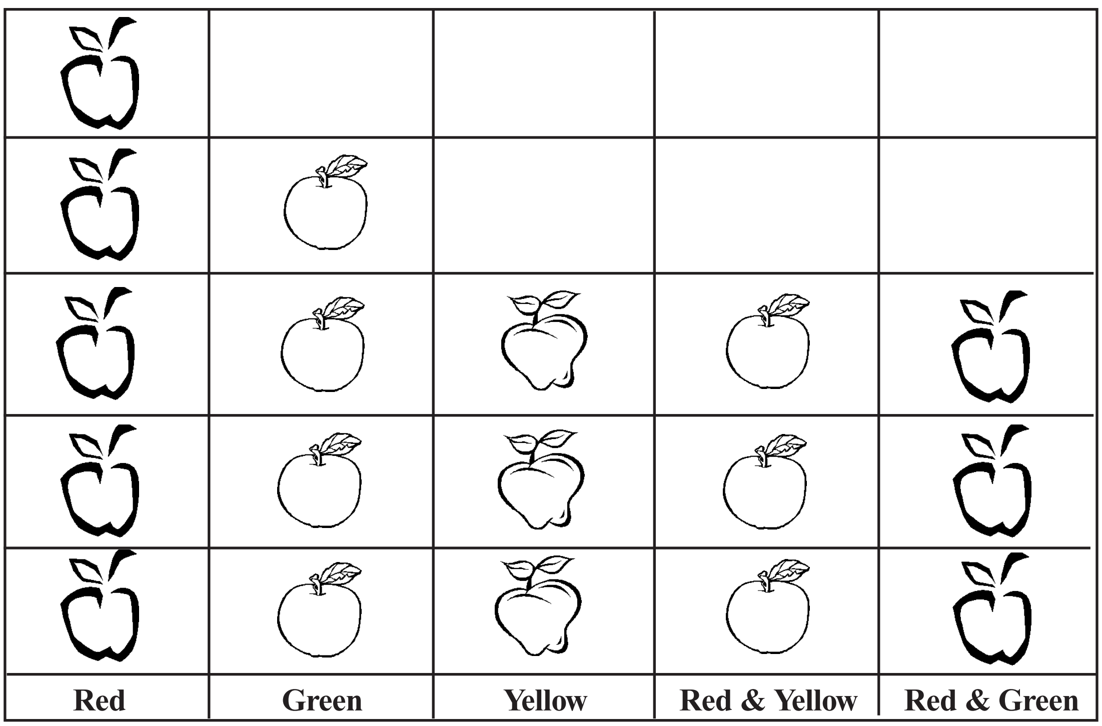
Types of Graphs

Object Graph:
This type of graph uses the actual object to display frequency (number).

Example: The teacher asks every student in the class to bring an apple to school. The teacher creates a blank grid on a large sheet of paper on the floor. With the help of the students, the grid is labeled to represent the types of apples they brought. The labeling could be by color, type of apple, taste, etc.

The apples in this activity would be real apples, not picture representations.

Types of Apples Brought by Ms. Sees' Class

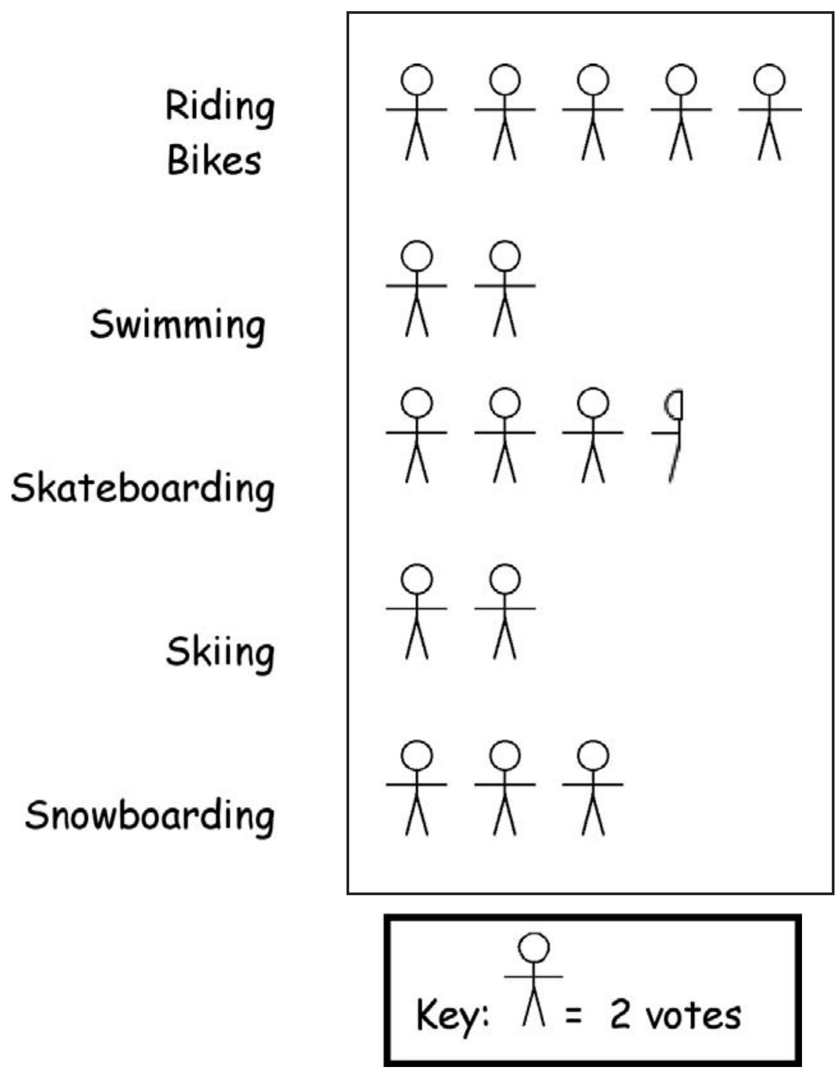


Types of Graphs (continued)

Pictographs or Picture Graphs:
This type of graph uses pictures or symbols to represent and compare data.

Example: A teacher asks her class to poll each other regarding their favorite recreational activity. The results may be displayed with a picto- or picture graph.

Favorite Types of Winter Recreation



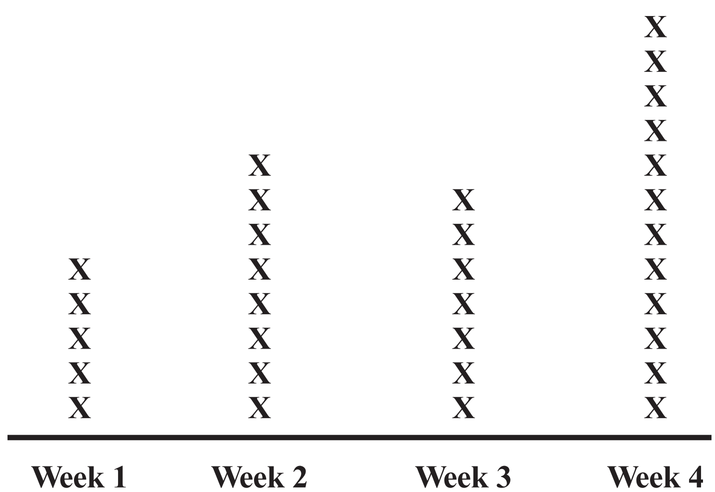
Types of Graphs (continued)

Line Plots:
Line plots are used to show frequency and can show comparison of data.

Example: A reading teacher asks her small group to keep track of how many books they finish per week for one month. The teacher totals the number of books read each week. The teacher enters the data into a simple table.

Weeks	Books
Week 1	5
Week 2	8
Week 3	7
Week 4	12

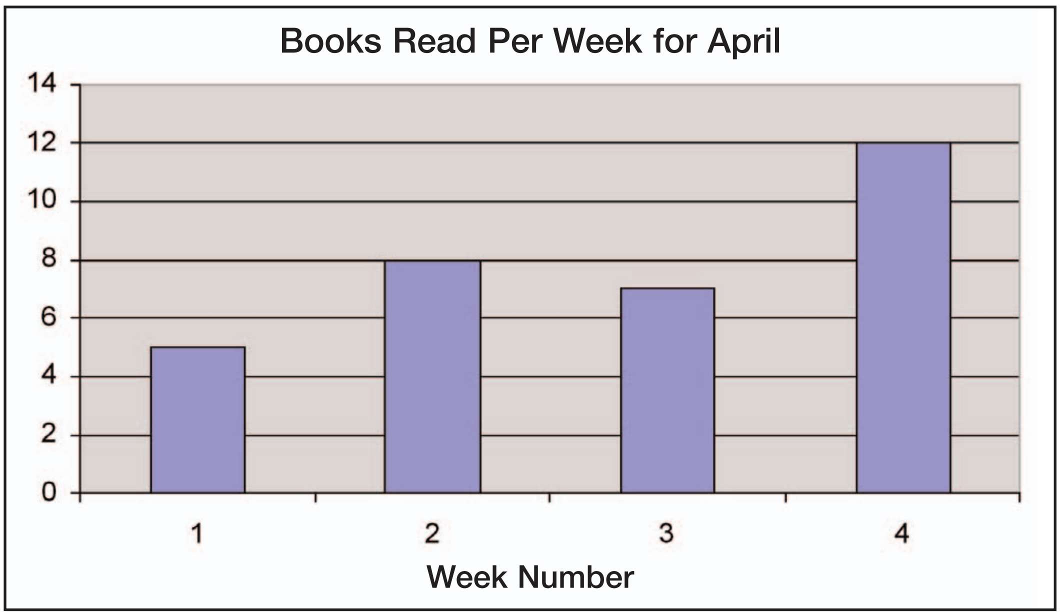
Students can plot the data from the table using graph paper to create a line plot to more graphically show the spread or range of data. Line plots are easy to create but can be time consuming for large data sets.



Types of Graphs (continued)

Bar Graphs:
In a simple bar graph the length of the bars represents numbers (frequency).

Example: The data from the data table representing the number of books read in one month can be used as an example of the use of the bar graph.



Types of Graphs (continued)

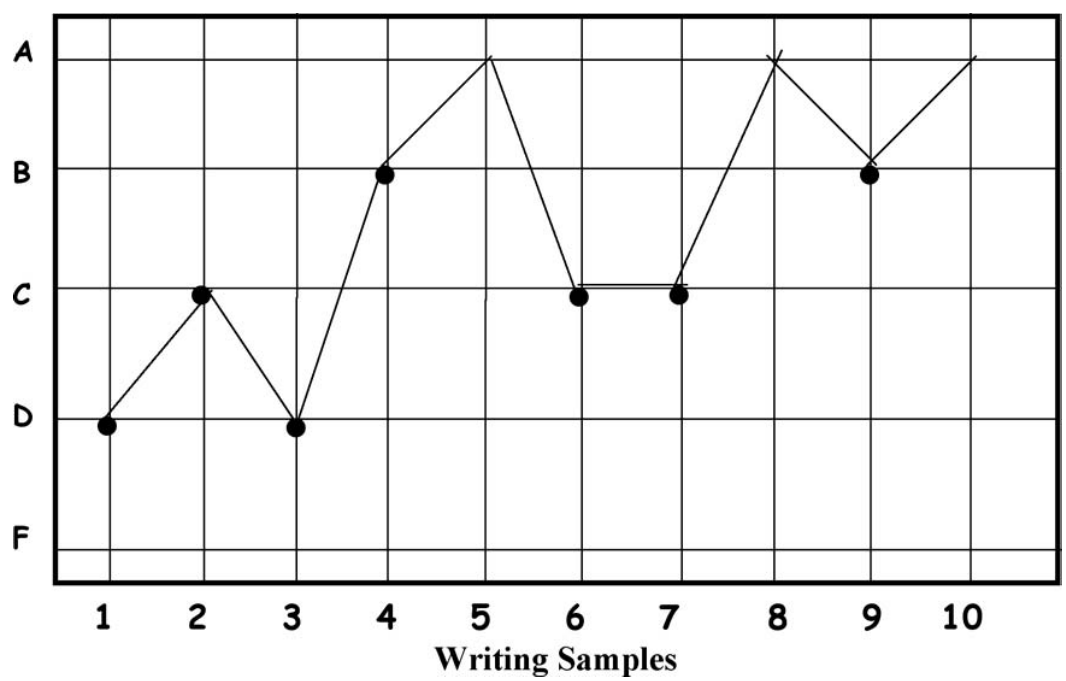
Line Graphs:
Line graphs show trends or changes in the data over time.

Example: A teacher would like to demonstrate to her 6th-grade student that he has made significant change in his writing skills in a more graphic way than just showing him his grades. She enters the data from her grade book in a line graph for 10 writing samples.

Grade Book
Timothy's grades

Writing Samples:	1	2	3	4	5	6	7	8	9	10
Grades:	D	C	D	B	A	C	C	A	B	A

Timothy's grades plotted on a line graph

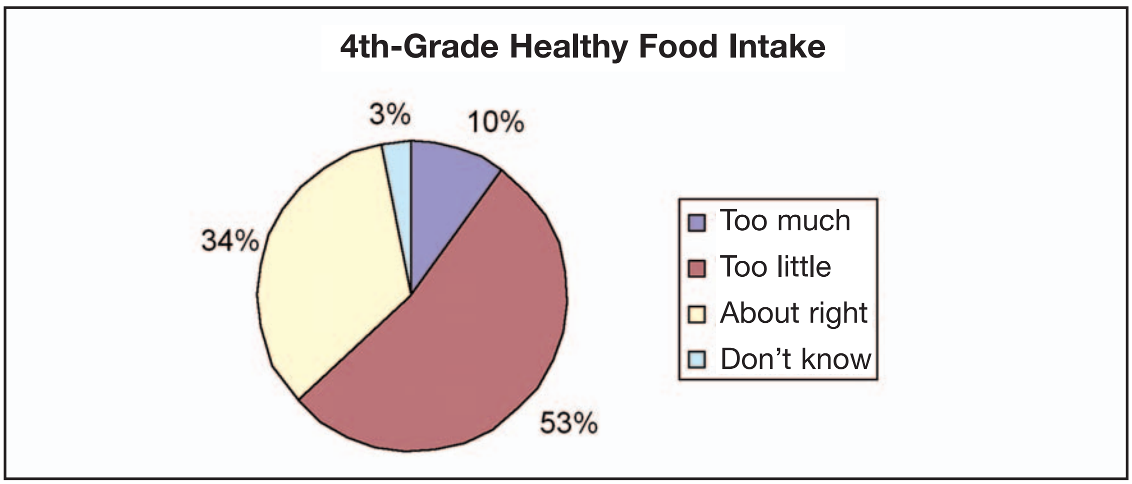


Types of Graphs (continued)

Circle Graphs:
Circle graphs, sometimes called pie charts, are a helpful way to visualize and organize data.

Example: A 4th-grade teacher asked her class to poll all 4th-grade students in the school regarding healthy foods. Students responded, and the data were put in a table that indicated intake of healthy food across an average day.

Too much	Too little	About FRight	Don't know
10	53	34	3



Graph Paper

Probability

Equally likely:

There is equal probability (chance) for an outcome.

Fair:

Object or event has options that are equally likely.

Likelihood:

How probable or likely it is that something will happen.

Likelihood is also known as **probability**.

Impossible: Event will never occur.

Certain: Event will absolutely occur.

Spinners

Directions for making a paper spinner with a paper clip:

- Straighten the paper clip except for the shortest hooked end. Example:

- Put a pencil point on the dot in the center of the circle.
- Place the spinner hook around the pencil point.
- Spin.

Activity 1:

1. Create a spinner that uses the numbers 1-4, where the number 1 is the most likely.

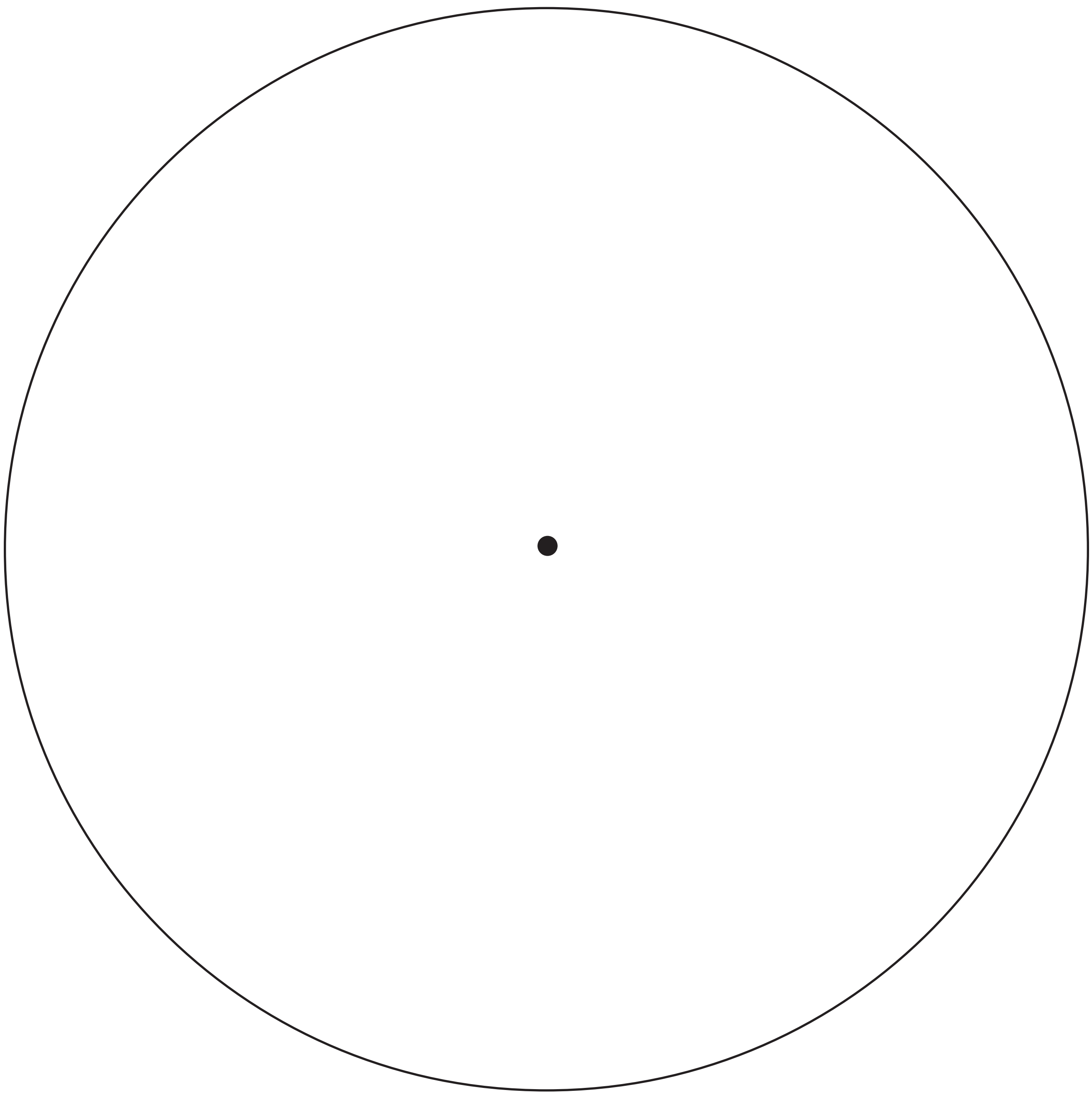
2. Record the results of your spins. Spin a sufficient number of times to be able to state that the results are most likely for use with this spinner.

3. Were the results of the spinning what you had predicted?

4. If not, make changes to the spinner until the results are those given in Step 1.

5. If you had to make changes, what were they?

Spinners (continued)



Spinners (continued)

Activity 2:

1. Create a spinner that is fair for the numbers 1-4.

2. Record the results of your spins. Spin a sufficient number of times to be able to state that the results are most likely for use with this spinner.

3. Were the results of the spinning what you had predicted?

4. If not, make changes to the spinner until the results are those given in Sep 1.

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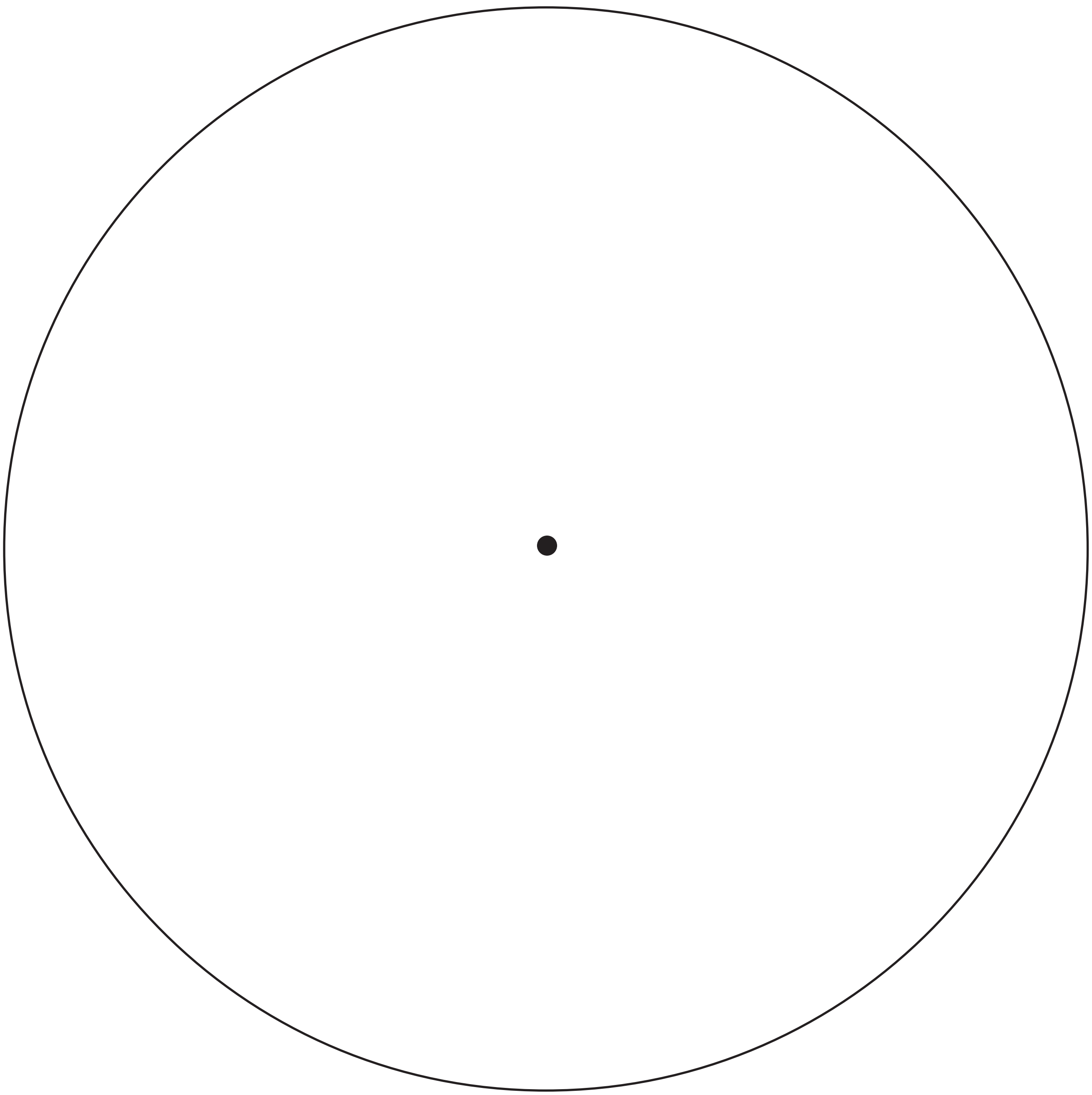
Spinners (continued)



Spinners (continued)

Activity 3:

1. Create a spinner that makes 3 an almost impossible result for the game.

2. Record the results of your spins. Spin a sufficient number of times to be able to state that the results are most likely for use with this spinner.

3. Were the results of the spinning what you had predicted?

4. If not, make changes to the spinner until the results are those given in Step 1.

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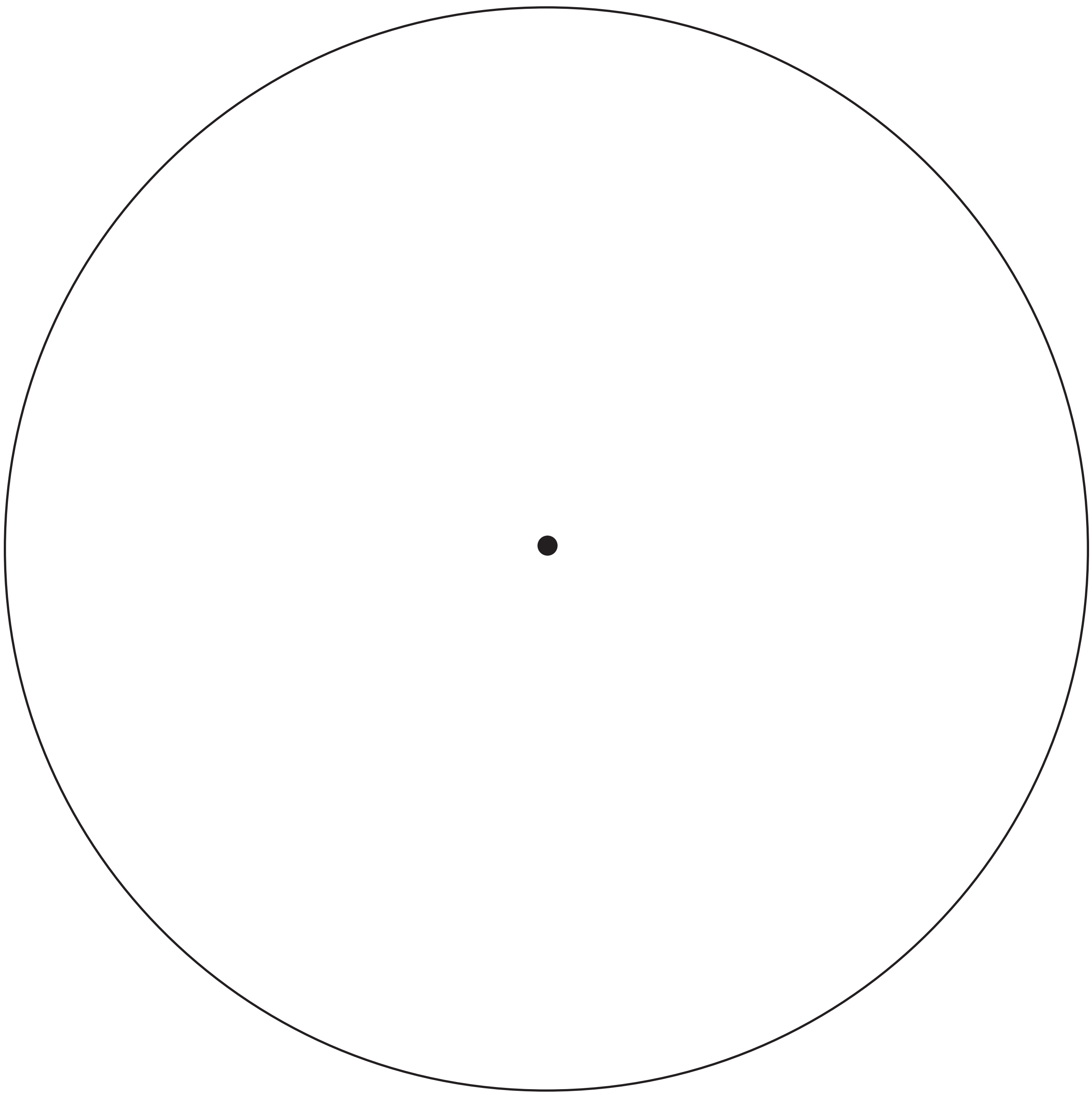
Spinners (continued)



Assignment 1: Seeing Graphs

Name: ____

Date: _________________________

The following assignment is worth **125 total points.**

The focus of the assignment is the use of tables and graphs to represent data. You will be finding real examples of graphs and analyzing your examples.

There are three steps to this assignment.

Step 1: (10 points)
Find an example of a graph in the newspaper, a magazine, the Internet, or some other source. Make a copy of the source to attach to the assignment.

Step 2: (80 points)
Answer the following questions about your graph.

1. Name the type of graph from the **Graph Types** handout that matches your example.

2. Give the title of your graph.

3. What kind of information is your graph measuring?

4. List the key (legend) or list the labels for the concepts on your graph being measured (vertical and horizontal direction).

5. Is it easy to read and understand? If not, provide examples of what you would do to make it easier to read.

Assignment 1: Seeing Graphs (continued)

Step 3: (35 points)

Write five questions about your graph that could be answered by students. Provide an answer key for your questions.

Module B Transparencies

Module Goals
Module B: Patterns

The paraeducator will:

1. Use concrete materials to aid pattern recognition and generalization

2. Relate basic patterns to algebraic concept development

3. Describe patterns and other relationships to interpret data using tables and graphs

4. Employ strategies of problem-solving to make predictions and determine the likelihood of an event

Patterns All Around Us

-
-
-
-
-
-
-
-
-

Patterns

Patterns are usually the first mathematical concepts children experience. Children typically recognize patterns as early as 3-1/2 years old.

A child's ability to:

- recognize,
- replicate,
- create,
- describe, and
- generalize patterns

is a key concept to developing later mathematical skills.

Repeating patterns:

Elements in the patterns repeat according to some rule. Repeating patterns are an easy place to begin helping children with prediction skills.

Growing patterns:

Elements in the patterns change (grow) according to some rule. These are also natural patterns, and are the introduction to algebraic concepts. The simplest growing pattern is counting.

Algebra

The ability to classify and sort objects by rules is the beginning of algebraic concepts.

Algebra is a method by which to describe and predict the behavior of a set of data.

Data can be anything: blocks, toys, weather, numbers, etc.

Algebraic thinking starts with patterns.

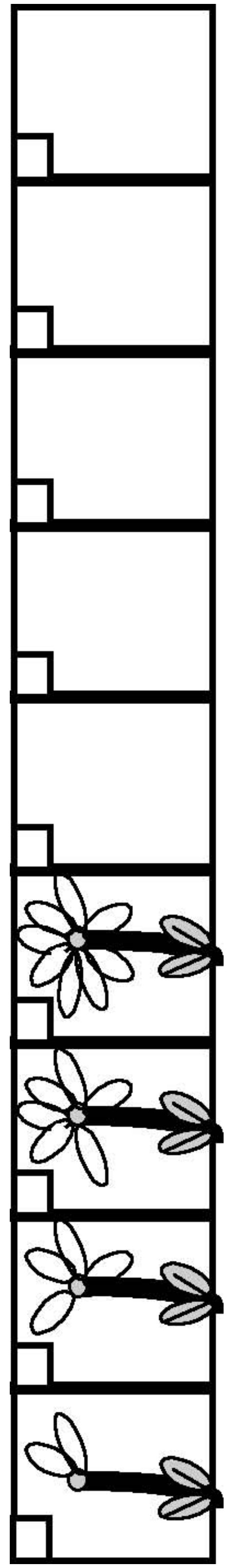
Variable

- **Is a symbol or object that can change value depending on the problem where it appears.**

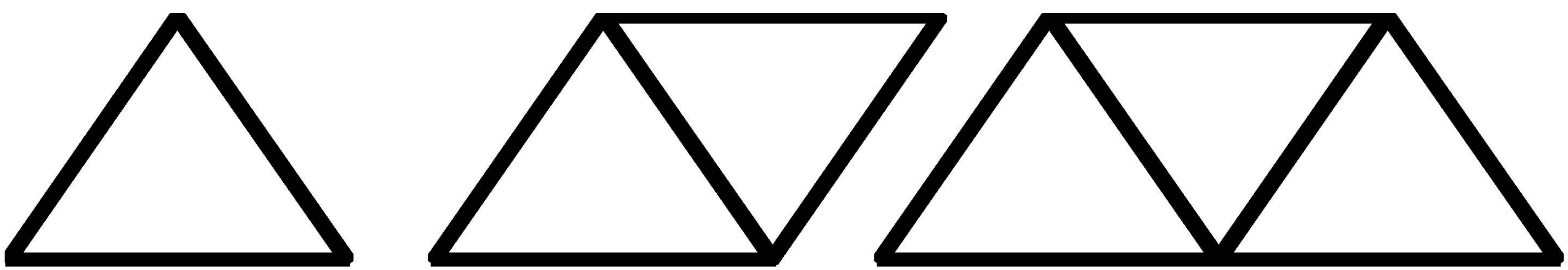
- **Simply holds a place until is has a value, then it changes with the next problem.**

- **Its value can vary or is "variable."**

Grow a Pattern



Troublesome Triangles



Tables and Graphs

Tables

Tables are used to organize data. Organized data are important in pattern recognition and developing algebraic expressions. Using a table is a way to *record frequency (number)* to show how often an item, a number, or a range of numbers occurs. Tables help *organize, summarize,* and *simplify*.

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Graphs are another mathematical tool for organizing data. Graphs are easily integrated throughout the elementary curriculum. For example, graphs can be used to show the number of students present daily, temperature changes throughout the day, or the *frequency* (number) of an attribute such as the number of students who have dogs. Graphs can be used for a one-time study or a study that looks for trends. Graphs are simply a way to help us picture data.

Types of Graphs

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The apples in this activity would be real apples, not picture representations.

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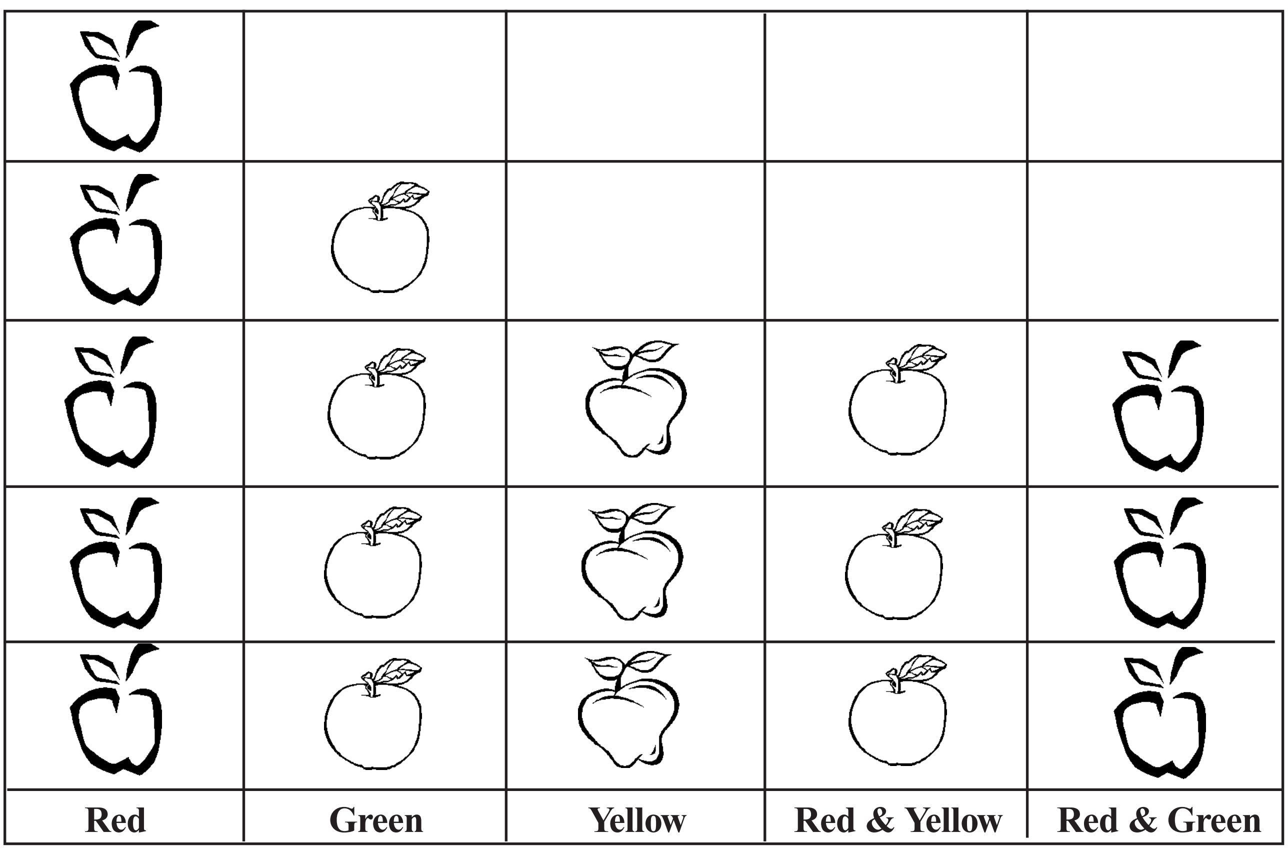

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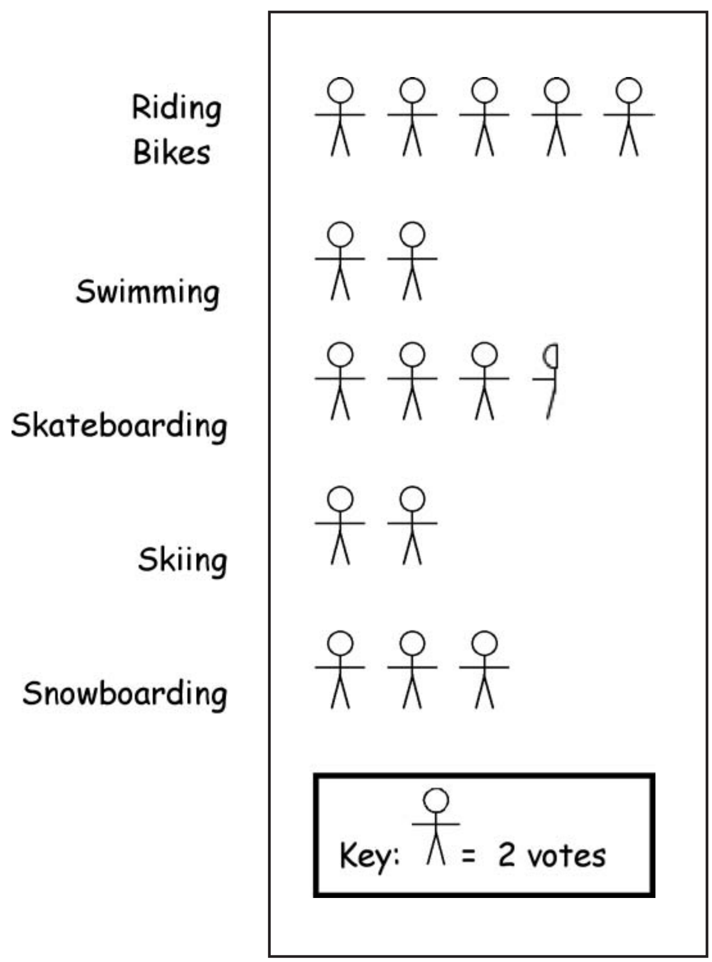

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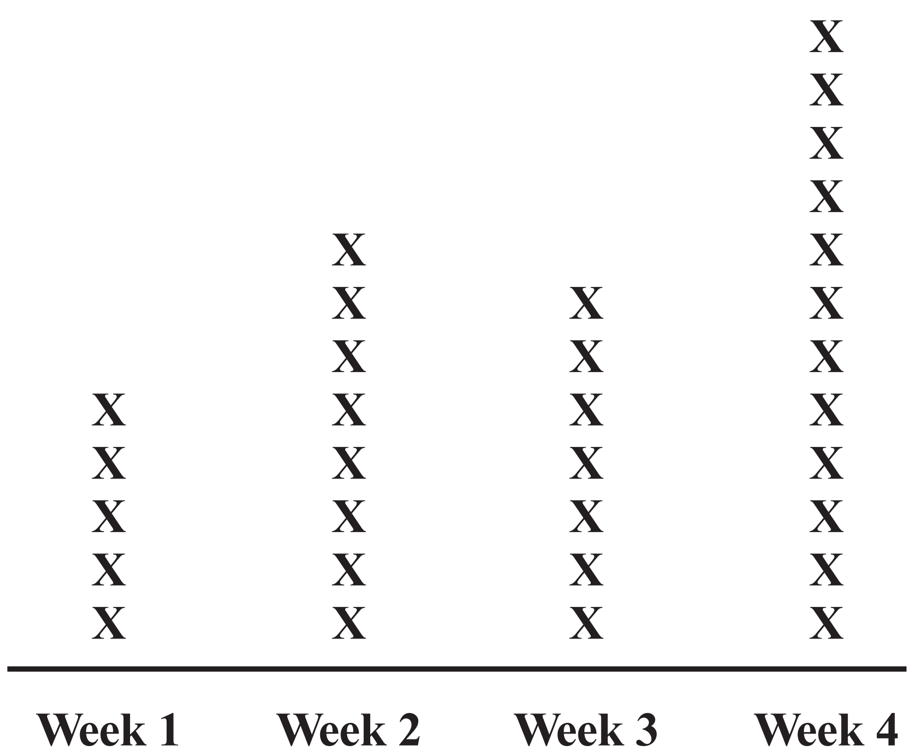

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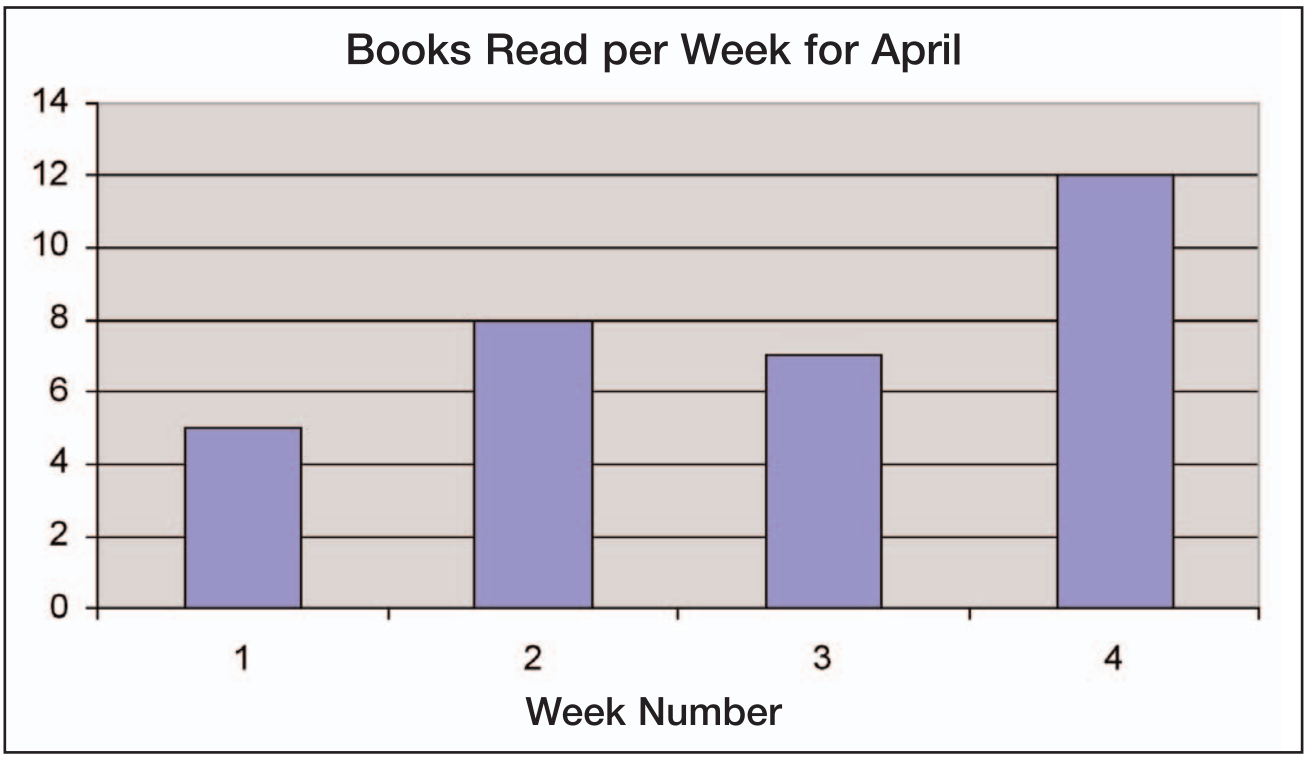

Types of Graphs

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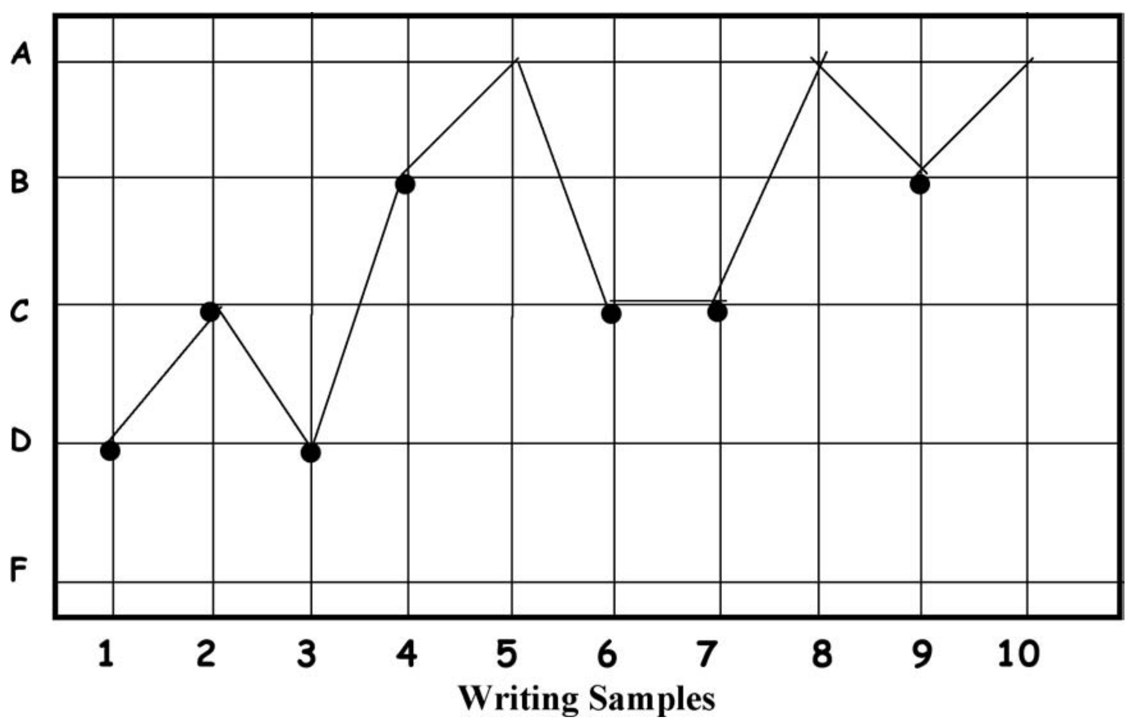

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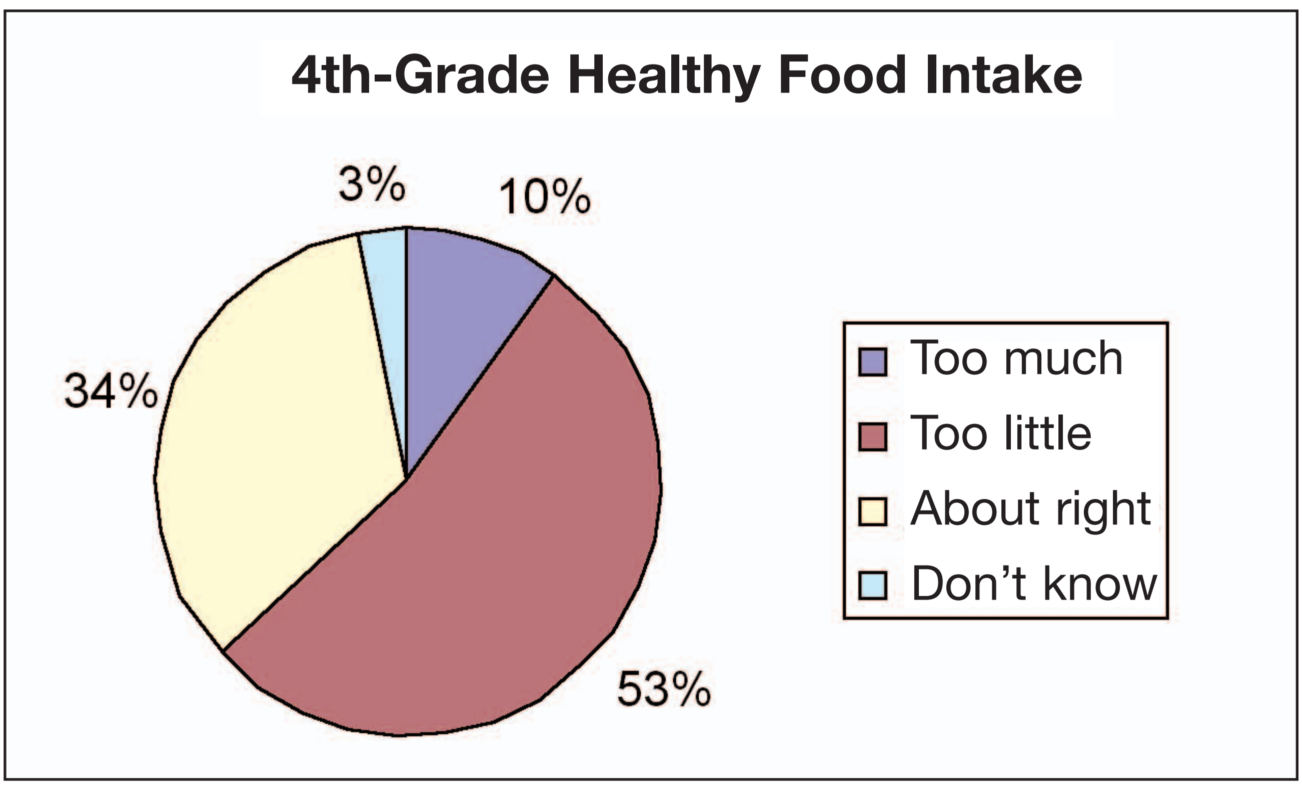

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Module C
Instructor's Guide

Module C:
Number Representation and Manipulation

A. Lecture: Module Goals
Use the **Module C Goals** transparency and handout **(T1/H1)** to review the module goals.

Module C: Number Representation and Manipulation
The paraeducator will:

1. Use multiple models to develop understandings of place value and the base-ten number system
2. Understand the meaning, effects, and relationships of the basic mathematical operations
3. Use patterns to explore algorithms for basic mathematical operations
4. Enhance number sense by building estimation skills
5. Define and communicate scenarios for appropriate use of basic operations (applications, money, time, etc.)

Goal 1: Use multiple models to develop an understanding of place value and the base-10 number system.

1.1 Lecture: Place Value Concepts

Note to Instructor: Remind participants to use their math journals to take notes during lecture periods.

The previous module focused on pattern recognition and algebraic concepts. These concepts build a solid foundation for further development in mathematical thinking and mathematical skills. These early skills are easily built upon. Students begin to see relationships and connections more clearly. That is, once students begin to look for patterns, concepts taught in mathematics become connected rather than isolated.

Early elementary skills focus on developing number concepts. Use the **Building the Foundation** handout and transparency **(H2/T2).**

Building the Foundation
The foundation of all number concepts begins with ***place value***.

In simpler terms, “place value” means:

The value of a digit (how much it stands for) is based on where it is “placed” in a number sequence. Our number system is based on a simple pattern of tens. It is called the *base-10 system*.

Place value concepts allow us to represent and manipulate small and large numbers. Early skills include counting and grouping numbers. Once students have opportunities to practice with grouping and counting, they find that they can quickly "tally" numbers based on the patterns they begin to see. The base-10 system is similar to a tally system. Use the **Tally** transparency **(T3).** When students learn to tally, they learn that at every 5, we cross the group of 4. Students quickly realize that this makes counting easier than counting by ones. The tally system is a base-5 system.

1.2 Activity: Place Value

1.2.1 Steps

- Use the chalkboard to review the place value of numbers. Review ones, tens, hundreds, and thousands place.
- Write several 4-digit numbers on the board and ask class members to identify the digit in each place: ones, tens, hundreds, and thousands.

1.3 Continued Lecture: Place Value Concepts

Early place value skills include:

- Counting
- Patterns
- Grouping numbers

When students tally and group numbers, they are using patterns. In base-10, each digit can only have a value of 0-9. In base-10, we trade in at every group of 10 and document that change by increasing the digit place by 1. Place value describes how many groups of 10 compose a particular number.

Place value practice enhances students' counting skills. Children begin to look for patterns to make it easier and quicker to count large amounts of ones (units). Helping children visualize the number patterns and the processes taking place gives them mental references as the concepts increase in complexity.

The following activities are designed to increase understanding and comfortable use of basic patterns related to place value and the base-10 number system.

1.4 Activity: Make It Count

Paraeducators will use patterns to count large numbers and build place value concepts.

Materials:

- Clear counters for use with overhead projector
- Counters or blocks for class use

1.4.1 Steps

- Place 24 counters on the overhead (with the overhead light off).
- Spread the counters around on the overhead.
- Turn the light on and ask class members to count the counters. Give them only a few seconds to do this and then turn the light off, or cover the counters.
- Ask how many counters there were. The hope is that no one was able to count them and, therefore, no one has the correct answer.
- Rearrange the counters on the overhead. Put them into two rows of 10 and one row of 4, or four groups of 5 with one group of 4.
- Turn on the light and ask them to again count.

After completing the activity with counters, discuss the following:

1. Which group was counted the most quickly and accurately?
2. Why was this so? Review the fact when the counters were grouped, each group had a specific ***value,*** so only the group had to be counted, not the individual counters in the group
3. Use the transparency **Grouping Data (T4).** Remind the class that grouping data makes counting easier and is one of the foundations of our number system.

Grouping Data

Grouping data makes counting easier and is one of the foundations of our number system.

1.5 Activity: Counting on You

Paraeducators will complete activities that encourage them to reflect upon their ability to count using groupings.

1.5.1 Steps

- Use transparencies **Cards, Dice**, and **Dominoes 1 (T5, T6, T7).**
- Show one picture per transparency at a time, covering all but the one being used.
- Ask participants to quickly count the number of objects on each of the cards, dice or dominoes.
- Point out how easy it is to determine the number of diamonds or dots because of how they have been grouped.
- Ask if anyone in the group counted the diamonds or dots one at a time. (Hopefully no one will have done that.)
- Explain that young children need to use simple games and activities such as cards, dice, and dominoes to reinforce their use of patterns, grouping, and counting. Remind paraeducators that some students need to be reminded to use their grouping skills when it is appropriate for counting.

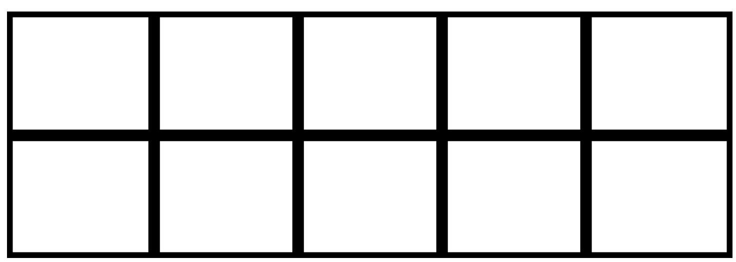
- Next, show some patterns that are "out of shape," and thus not easily countable.
- Use the transparency **Dominoes 2 (T8).** Show one domino at a time briefly, and ask participants to count the dots.
- Direct class members to their math journal section *Reflections* and give them time to reflect on their experiences about counting and grouping. Use the following as suggestions to spark their reflection and writing:
  - ▲ What was the difference between the first and second group of dominoes?
  - ▲ Which was easier to count? Why?
  - ▲ How could you use this information to help students in mathematics?
  - ▲ When students use expected patterns to help them count, what is the outcome?

1.6 Activity: "Give Me Ten"

Paraeducators will use the ten frame to show place value concepts to prepare for arithmetic topics.

Materials:
- At least 100 counters (beans, discs, blocks, etc.) per pair of class members
- **Ten Frame** handout and transparency **(H3/T9)**

A ten frame is simply a grid with 10 open frames to use for grouping activities.



1.6.1 Steps

- Divide the class into pairs. Give each participant the **Ten Frame** handout **(H3).** Use the transparency **Ten Frame (T9).**
- Have partners use one handout and as many ten frames as necessary to show "25" with counters.
- Have pairs use the other partner's handout and counters to show "52."
- Then, have the partners within the pairs compare their ten frames.
- Ask what happened as individual frames were filled:
  - ▲ Answer: Moved on to fill the next frame
- Discuss the similarities and differences of the two numbers:
  Possible responses:
  - ▲ Both have a 2 and a 5

- ▲ 52 fills in more of the ten frames
- ▲ 52 has 5 groups of 10 and one frame with only 2
- ▲ 25 has 2 groups of 10 and one frame with only 5
- ▲ One way for a student to determine that the number 52 is larger is that it covers more space
- Analyze each number carefully, noticing the place of each digit.
  Possible responses:
  - ▲ For 52, 5 full frames and 2 extras
    - o 5 shows groups
    - o 2 shows extras
  - ▲ For 25, 2 full frames and 5 extras
    - o 2 shows groups
    - o 5 shows extras
    - o Groups (tens) are always to the left of the singles
    - o Singles (units or ones) are always on the right
- Work on vocabulary.
  - ▲ A ten frame represents a group of 10
  - ▲ For 52, there are 5 groups of 10 (or 5 tens) and 2 singles (2 ones or units)
  - ▲ For 25, there are 2 groups of 10 (or 2 tens) and 5 singles (5 ones or units)
- Working in pairs, have participants choose numbers to build using the **Ten Frames** handout.
  - ▲ Describe the process in detail
  - ▲ Discuss how many ones and tens
- Share that similar activities can be done with straws (rubber banding groups of 10) or money.
  Note: Many classrooms use Cuisenaire rods or similar materials. Consequently many children learn to think of 10 as a long strip or rod (known as a long or skinny) and the 100 as flat or a 10 x 10 square due to base-10 blocks.

1.7 Activity: What's the Number?

Paraeducators will increase their knowledge of place value by looking at the effect of number combinations on the magnitude of the resulting number.

Materials:

- Deck of playing cards (discard face cards and jokers)
- Base-10 blocks (or paper pieces) – per pair
  ***Note to Instructor**:* If you do not have base-10 blocks available, use the **Paper Base-10 Blocks** handout **(H4)** and cut out enough of the paper blocks for your class.
- Handout **What's the Number? (H5)**

1.7.1 Steps

- Divide the class into pairs.
- Define blocks:
  - ▲ Flat = 100

- ▲ Long or skinny = 10
- ▲ Unit = 1
- Draw three cards from the deck and show them to the class. Example, you may have drawn a 5, 7 and a 2.
- Direct the pairs to complete the questions on the **What's the Number?** handout. As the pairs work on the activity, work along with them, modeling the numbers on the overhead.
- Working in pairs, complete the following activity: Using the numbers of the cards that the instructor drew from the deck:

1. Show/write the smallest number you can make with the cards *(using the example numbers, the smallest number would be 257).*
   - ▲ The instructor models the number with the blocks and the pairs draw it on their handout (2 flats, 5 longs, 7 units).
2. Show/write the largest number you can make with the cards *(using the example numbers, the largest number would be 752)*.
   - ▲ The instructor models the number with the blocks and the pairs draw it on their handout (7 flats, 5 longs, 2 units).
3. Using the blocks, the pairs model any number between the largest and smallest (*different pairs can model different numbers, i.e., 275, 527, 572, 725).*
4. How many different numbers can you make with these three cards? *(if the cards are all different, there will be a combination of six numbers, for example: 257, 275, 527, 572, 725, and 752).*

- Direct the class to generate additional questions for the group to answer using other math terms such as *almost, between, at least*, etc. Examples:
  - ▲ *Show me a model of a number that is near or close to 752.*
  - ▲ *Show me a model of a number that is between 460 and 465.*
  - ▲ *Show me a model of a number that is less than 322.*
- Discuss the results of this activity as necessary to ensure understanding.

Goal 2: Understand the meaning, effects, and relationships of the basic mathematical operations.

2.1 Lecture: The Math Family

With arithmetic, we deal with four basic operations: addition, subtraction, multiplication, and division. While often taught separately, these are actually all related concepts. These basic operations are simply ways to combine numbers. Use the **The Math Family** handout and transparency **(H6/T10)** as you discuss the individual processes.

The Math Family

Addition is generally the first concept taught. Basic addition concepts come from counting. Mathematics in K-4 builds significantly on counting skills to master basic facts and arithmetic processes. **Addition involves combining objects/numbers (*addends*) into a larger group (*sum*), increasing the group size from either original group.** The order of numbers does not matter for addition to produce the same answer. There are 100 basic addition facts using the inverses of the addition facts 0 through 9.

Subtraction generally follows addition instruction. Subtraction is the inverse (opposite) of addition. **Subtraction starts with a larger group, and then some of the group is removed, producing the *difference* between the two original numbers**. The answer for subtraction is smaller than the starting number. Subtraction may be taught as counting backward or counting forward as in addition. There are 100 basic subtraction facts.

Multiplication is short-hand for addition. Every basic multiplication fact can be rewritten as addition. This is important as students learn basic facts. They also need to see that order does not matter, which allows them to reorder numbers to utilize simpler counting patterns. **Multiplication, like addition, combines two numbers (*factors*) and produces a larger number (*product*).** There are 100 basic multiplication facts.

Division is generally the final basic concept taught. Just as addition and multiplication are related, so are subtraction and division. **Division starts with a large number and is partitioned or divided into a specified number of groups or groups containing a specified number**. This is similar to subtraction in that **the answer (*quotient*) is a smaller number than the beginning amount**. Division is repeated subtraction. Division can also be taught in terms of multiplication facts. There are only 90 basic division facts because there are no division facts with zero as the *divisor* (number that you divide by).

2.2 Activity: Patterns with Counting

Paraeducators will participate in activities that reinforce seeing patterns in counting.

Materials:

- Cutouts in rectangular or "L" shapes that will adequately cover portions of the 100 board when used on the overhead (or use squares to cover patterns)
- The transparency **100 Board (T11)**

2.2.1 Steps

- Have participants continue to work either in pairs or individually.
- Remind class members that counting is a fundamental component of mathematics and that patterns can be used to help with counting skills.
- Use the transparency **100 Board (T11).**
- Turn off the overhead and place a cutout on the overhead covering up several numbers.
- Turn on the overhead and ask the class to fill in the missing numbers.
  - ▲ Ask them how they knew; that is, what skills were necessary to identify the missing numbers (*counting, familiarity with the number sequence so that the missing numbers are easily identified*).
  - ▲ Discuss what patterns are necessary in a single row *(counting by ones, keeping the tens place the same).*

- ▲ Discuss what patterns are necessary going down a column (*counting by tens or changing the tens numbers but keeping the ones place the same).*
- Remind the class that vocabulary is very important in communication – students should be asked to explain how they got their answer, which can be challenging.
- Do several examples, covering different numbers in different patterns such as three numbers in a column, "Z shapes," diagonals, etc.
- Remind participants to take notes in their math journals and to reflect upon how they could use this sort of activity to support young students.

2.3 Lecture: Addition and Subtraction

The previous activity of Patterns with Counting set the stage for **addition** fact families. Use the **Addition Facts** and **100 Addition Facts** handouts and transparencies **(H7, 8/T12, 13)** during this discussion. Put the **Addition Facts** transparency up and ask class members to refer to their handout. Direct them to use their math journals and to write about the patterns they notice about addition facts.

Noting patterns is an important part of learning basic facts. If students do not see patterns, facts can seem overwhelming. After working individually at first, as a group, look at the organization of the facts that are given. Discuss what patterns are obvious from the set-up of the handout. Some possible patterns are the first row is 0 facts, each row loses a problem at the front, or diagonal facts are *doubles* (same addends).

Pattern usage for higher facts comes from experience with obvious patterns with 0, 1, and doubles. As a group, fill in the zero row. The pattern should be clear that adding zero does not affect the number.

Discuss the reason for the missing fact of 0+1 in the second row. The reason is that 1+0 gives the same answer as 0+1. This is the ***commutative property of addition***. Changing the order in addition to get the same answer is an important concept as students look for the easiest way to solve a problem. Seeing a pattern and remembering it makes it easier.

Complete the second row of "1+ ___." Note how quickly this can be accomplished because adding 1 is just like counting by 1 more.

Find the *sums* (answers to addition problems) of the diagonal facts. Discuss what makes these facts special. These ***double*** facts (same addend) are easily understood (and memorized), and act as a starting point for higher facts. For example, if 8+8=16, then 8+9 can be easily calculated as 1 more than the double fact without going back and counting from the beginning.

The commutative concept of addition helps students make choices about how to approach a problem. With a complex fact such as 5+6, one must decide whether it is easier to see the problem as six ***counting on*** five more, or five ***counting on*** six more.

Memorization comes faster when students realize that 5+6 and 6+5 result in the same answer. Until facts are memorized, children must have some approach to finding reason in this process.

Fill in the missing facts on the **Addition Facts** handout. Discuss other patterns that could help students master their facts. Use the **100 Board** transparency **(T11)** again to help see patterns. Columns are counting from the first addend down the column, increasing by one. Other diagonals going left to right are the same sum. Although there are 100 addition facts, many are simply a repeat of these basic facts. Knowing this helps cut down on the need to memorize.

An additional strategy to ***doubles*** and ***counting on*** is the ***making tens*** strategy, which links back to place value concepts. Students quickly learn that 10 is an easy number to "add on" to. Using concrete objects, like the base-10 blocks, students can easily regroup to 10 and then add on the extra pieces. This process involves some higher-level planning, but is effective in later problem-solving.

Direct participants to use their base-10 blocks to demonstrate:

- 8+5= (use objects or blocks with 8 and 5)
- Regroup into 10+3=13

Subtraction can be taught and supported in the same manner. A chart will be made as a homework assignment for the basic subtraction facts. Subtraction is not commutative; the largest number must come first until integer concepts are taught in later grades. Fact memorization can be reduced by making students understand that addition facts support subtraction. For example, we know the problem 5+9=14. This tells us that two problems can be created: 14–5=9 and 14–9=5. Spend significant time looking at these relationships with students.

Many children struggle with subtraction because ***counting back*** is difficult as they do not know whether to include the original number in the process. For many students, counting forward is easier. Using the problem above, a student could count 13, 12, 11, 10, 9 to get the answer for 14–5, or could count forward 6, 7, 8, 9, 10, 11, 12, 13, 14, equaling 9. The most appropriate strategy depends on the student.

Use concrete objects and number visualization (such as dice patterns) with all of these strategies so that students have a mental picture of the problem. Memorization is only effective if students have an underlying understanding of the concepts. Students who memorize without understanding have difficulty recalling facts.

2.4 Activities: Pattern and Basic Facts

Paraeducators will use the 100 board to compare patterns for a variety of counting rules that assist in generalizing mathematical algorithms for basic computation.

Materials:

- A copy of the handout **100 Board (H9)** for each participant
- **Patterns and Basic Facts** transparency **(T14)**
- Crayons (or other coloring tools)

2.4.1 Steps

- Create groups of 4-5.
- Within the groups, direct ***each*** paraeducator to color his or her own board, creating a chart according to a particular rule:
  - ▲ Counting by 2
  - ▲ Counting by 3
  - ▲ Counting by 6
  - ▲ Counting by 5
  - ▲ Counting by 10
  - ▲ Choosing any related rules for comparison
- After everybody is finished, ask them to compare their charts within the group. Direct them to record their observations in their journals under *Problem-Solving* or *Reflections*. Use the **Pattern and Basic Facts** transparency **(T14)** to provide direction.

Pattern and Basic Facts

Give participants the following directions:

- After you have colored the pattern of your choice on your basic facts handout, work with your group, looking for:
  - ▲ similarities and differences in patterns
  - ▲ numbers that are colored on more than one board
  - ▲ numbers that are seldom colored

- Record all observations in your journal *Reflections* section.
- As a class, debrief by sharing observations.
- Brainstorm what possible operations/skills could be taught using this exercise. The following discussion will also include information that may be covered in this debriefing.

2.5 Discussion: Interpreting 100 Board Results for Multiplication and Division

Materials:

- Copies of 100 boards used in the previous activity
- Transparency and handout **Graph Paper (T15/H10)**

Multiplication strategies are similar to addition strategies; multiplication is repeated addition. Before memorizing basic multiplication facts, students must comprehend what it means to multiply. Use the following steps of guided discussion:

- As a group, use the **Graph Paper** handout and ask each class member to draw or indicate the quantity "12" in a rectangular formation or ***array***.
- Ask participants to hold up their drawings. There should be a variety of arrays, such as, 3x4, 4x3, 2x6, 6x2, 1x12, and 12x1.
- Point out that if we wanted to know how many squares any given array contains, we could simply count. But that would take too much time for larger numbers, and there is always a high likelihood of making a mistake when counting.
- Duplicate a 3x4 array on the **Graph Paper** transparency. Discuss different methods for counting faster, including the following:
    - ▲ Redraw and group by 10 and then add on,
    - ▲ Write a repeated addition problem such as 3+3+3+3, or
    - ▲ Do 3x4. (3x4 may be thought of as counting by 3's four times or by 4's three times)

  Multiplication should be seen as a more efficient way to count.

2.6 Continued Discussion

Continue to guide the lecture and discussion using the following points:

- Referring back to the 100 board used in **Activity 2.4,** discuss what multiplication fact families are on the "counting by 2's" chart, such as the fact family for multiplying by 2.
- As a group, write some multiplication problems from the charts, such as 2x1 (2 square 1 time) = 2, 2x2 (2 squares 2 times) = 4, 2x3 (3 squares 3 times) = 6. This is the counting pattern 2, 4, 6, … or even numbers. Discuss the same issues for the other charts that small groups charted or drew in **Activity 2.4.**
- Discuss how repeated addition can stem from this exploration. From the 2's chart, 2x4 can be seen as 2 squares + 2 squares + 2 squares + 2 squares or 2+2+2+2.
- As multiplication also possesses the *commutative property*, this problem can also be written as 4x2 or, 4 squares + 4 squares, or 4+4.
- Critical discussions and practice with translating multiplication into groups and changing order are vital to comprehending multiplication.
- To see patterns in multiplication, list fact families in columns and predict the product.
  1x7 = 7 (1 group of 7)
  2x = 14 (2 groups of 7)
  3x = 21 (3 groups of 7)
  4x7 = 28 (4 groups of 7)
- Continue this pattern by predicting the next parts of the fact family.
- The pattern of adding 7 is very beneficial to solving more difficult problems such as 7x8. If students know 7x7 (double) = 49, they can be assisted to understand that 7x8 is just 7x7 with another group of 7 added on, or 56.
- Compare the 2, 3, and 6 charts used in **Activity 2.4**. Discuss numbers that are colored on all three sheets such as 12 (*product)*. Twelve has several *factor* pairs (numbers that are multiplied together) as listed above. Find other examples of

commonly colored squares (*products*) on other sheets. Using these types of exercises for comprehension and practice assists students in organizing the vast number of facts they need to memorize.

2.7 Continued Discussion: Division

Continue to guide lecture and discussion with the following points:

- Division works with multiplication and subtraction.
- Again, compare the 2, 3, and 6 charts. Division is the opposite of multiplication.
- Noting that 12 is on all three sheets or charts, it follows that 12 must have factors of 2, 3, and 6.
- The division problem can be solved by thinking about multiplication: what number times 2 gives the product of 12?
- Division can be visualized by making groups of 2. In this case there would be six groups of 2. This is usually easier in the beginning.
- Another way to interpret 12÷2 is to physically divide 12 into two groups. There would be six elements in each group.
- Just as multiplication can be repeated addition, division can be repeated subtraction. Working backward by subtracting also gives the missing factor, such as:

$$\begin{array}{r}
12 \\
-\ \ 2 \\ \hline
10 \\
-\ \ 2 \\ \hline
8 \\
-\ \ 2 \\ \hline
6 \\
-\ \ 2 \\ \hline
4 \\
-\ \ 2 \\ \hline
2 \\
-\ \ 2 \\ \hline
0
\end{array} \qquad \text{6 subtractions}$$

The question, "what is 12÷2?" can be explained by using the subtraction example shown above. The question really is "how many groups of 2 can be found in the number 12?" The learner then begins the process of subtracting groups of 2 from the number 12 until he reaches "0." The learner then notes that it took six separate subtractions of 2, so 2 can be subtracted from 12 six times. The answer then becomes: 12÷2=6.

Multiplication and division often depends on the success of addition and subtraction. Varied activities for visualization of these concepts are important prior to fact memorization. As students move toward algorithms, basic facts will be their foundation. **Memorization is vital for efficiency, but understanding is the key to mathematical development.**

Goal 3: Use patterns to explore algorithms for basic mathematical operations.

3.1 Lecture: Algorithms in the Classroom

As discussed in earlier modules, many people associate mathematics with arithmetic. Arithmetic is the mechanics of mathematics; that is, arithmetic includes the computations that occur in mathematics. This includes the rules and processes for solving a variety of computational problems. ***Algorithms*** are simply the steps of, or recipe for, each of the computational processes. The most common computational process or algorithmic processes are the ones we have just been discussing: addition, subtraction, multiplication, and division.

Algorithms are often overwhelming to young children because they have difficulty keeping the steps separate for different mathematical operations. Remember when division was introduced in elementary school and it seemed as if there were so many steps to remember just for a simple division problem? Algorithms have their place in mathematics. Without algorithms, we cannot complete more complex problems because we do not have the foundation. **Algorithms taught too early are detrimental to the mathematics process. Children learn to memorize without thinking through the process.** This is evident when a child is presented with a novel problem and is unable generalize her knowledge. This hinders problem-solving. Young students or struggling older students need much opportunity to practice the activities that support the foundations of mathematical thinking.

For students who struggle with these processes, finding a personal link to patterns and language may help them learn the concept rather than memorizing abstract rules. A student should never feel helpless when confronted with the opportunity to solve a problem; students need strategies to back them up if they forget a rule.

The following activities were written based on the assumption that students have already had hands-on experience with the underlying operations. These activities are intended to help learn the algorithmic process for the basic operations.

3.2 Activity: Clear the Boards

Paraeducators will use knowledge of place value and basic facts to explore addition and subtraction algorithms.

Materials:

- Base-10 blocks (or paper patterns)
- 2 dice per pair of students
- **Clear the Boards** handout and transparency **(H11/T16)** (1 per person)

3.2.1 Steps

- The following activity can be modified for addition or subtraction and can be carried with smaller numbers depending on grade-level expectations
- Review with the group the standard algorithms for addition and subtraction using the following:

5 4	3 7	3 7	5 0
+ 3 1	+ 2 3	+ 1 2	+ 3 7
5 4	3 7	3 7	5 0

Note to Instructor: Avoid the words “carry” and “borrow.” They are misleading, and are not the correct or formal terminology that paraeducators need to become familiar with and use with ease. Instead, use the terms “regroup” and “ trade in.”

- Play “Clear the Boards.” Use the following instructions for the game:
  - ▲ Use the transparency and handout **Clear the Boards (T16/H11).**
  - ▲ Divide class members into pairs.
  - ▲ Provide each pair with a pair of dice, the handout and base-10 materials (flats, longs and units).
  - ▲ Each player starts with a flat in the hundreds column.
  - ▲ The first player roles the dice separately.
    - o The first die represents numbers in the tens place
    - o The second die represents numbers in the ones place

      Ex. First = 1, second = 7; the roll is 17
  - ▲ Take away the rolled amount from the hundred. This requires trading in so that the problem is possible (see below for an explanation of this process).
  - ▲ Record the algorithm and check to make sure it matches what is left on the board.
  - ▲ Players take turns rolling and subtracting.
  - ▲ Once a player gets below 10, only one die is used on the roll.
  - ▲ The first player to clear the board wins.
- Emphasize that players must communicate about how the trade is taking place. It may be necessary to play the game several turns using the **Clear the Boards** transparency with the class observing the process before they proceed as pairs .
  - ▲ “I am trading in the flat for ten 10s.”
  - ▲ “I am trading one 10 for 10 ones because I have to take away 7 ones.”
  - ▲ “I can now subtract 17, which is 1 ten and 7 ones.”
  - ▲ “I have 8 tens and 3 ones left.”
  - ▲ “For my problem, I start with 100, which says I have 1 hundred and 0 tens or ones.”
  - ▲ “I cross out my 1 and make the next zero a 10 to show my ten 10s.”
  - ▲ “I cross out that 10 and make it a 9 for my 9 tens and make the last 0 a 10 to show my 10 ones.”
  - ▲ “Now I can take 7 from 10, which leaves 3 and another 10, which leaves 8.”
  - ▲ “I have proven that I have 83 left.”

3.3 Lecture: New Thoughts for Addition and Subtraction

The previous activity above is the traditional algorithm for subtraction and the related addition concepts.

Several other algorithms are less traditional but often work well for students who are struggling with algorithms. These methods may also be used to help understand the traditional algorithm. Some of them are as follows.

***Addition*:**

1. Use the **Place Value Charts 1** handout and transparency **(H12/T17).**
    - Review the process of addition using place value charts to show regrouping and "expanded notation" concepts for ease of carrying out addition.
    - Ask participants to use their math journals to take notes regarding the process and use the handout to record the process of adding using regrouping and expanded notation.
    - Use the example of 35+26.
    - Explain that "at 10 ones, we must regroup."
    - Young or struggling learners often make a common error: giving the final answer as 511. They also try to add from left to right, which is a strongly reinforced concept that may be carried over from reading.
    - The first step is to rewrite the problem into the chart by place value. Next, add the place value columns. As students place the 11 in the ones places, ask them to think about using manipulatives of single units (as when working with base-10 blocks). Ask them to think about how they would express 11 single units: They would replace 11 units with one "long" and one "unit." The same is true in the table below.

	Tens	Ones
	3	5
+	2	6
	5	11
	6	1

    - It is also appropriate to show the traditional algorithm notation. The one long becomes the number "carried" to the tens place in the place value chart.
    - Another example of a new thinking:
      It is easy to add zeroes:

Ex.
$$\begin{array}{r} 7\ \ 5 \\ +\ 2\ \ 7 \\ \hline \end{array}$$

Hundreds	Tens	Ones
	7	5
	2	7
	9	12
1	0	2

This example can also be expressed with expanded notation using zeroes to help with addition.

$$\text{Ex.}\quad \begin{array}{rcr} 70 & + & 5 \\ 20 & + & 72 \\ \hline 90 & + & 12 \end{array}$$

Hundreds	Tens	Ones
	9	0
		12
	10	2
1	0	2

- A common error with this approach is that it is often difficult to break a number apart and then adding it back together correctly.

2. Add left to right – another new thought, Example 1.
   - This also works extremely well with expanded notation, as it can be done as one problem (see below) or as separate problems by place value and then added.

     For example:

     568 + 751:

$$\begin{array}{rrcrcr} & 500 & + & 60 & + & 8 \\ + & 700 & + & 50 & + & 1, \\ \hline & 1200 & + & 110 & + & 9 \end{array} \qquad \text{then} \qquad \begin{array}{rr} & 1200 \\ & 110 \\ + & 9 \\ \hline & 1319 \end{array}$$

- With more practice, students can use regular place value as shown below.
- A common error with this approach is that students may not place the numbers in the appropriate column and then add wrong. For instance, if they were adding 8+1 in the ones place, instead of 8+7 (as in the second example below), they could become confused about where to place the number "9."

Example 1

$$\begin{array}{rrrr} & 5 & 6 & 8 \\ + & 7 & 5 & 1 \\ \hline 1 & 2 & & \\ & 1 & 1 & \\ & & & 9 \\ \hline 1 & 3 & 1 & 9 \end{array}$$

Example 2

$$\begin{array}{rrrr} & 5 & 6 & 8 \\ + & 7 & 5 & 7 \\ \hline 1 & 2 & & \\ & 1 & 1 & \\ & & 1 & 5 \\ \hline 1 & 3 & 2 & 5 \end{array}$$

Subtraction:

1. Use place value charts (use transparency and handout **Place Value Charts 2 [T18/H13]).**
   - It is similar to addition, except that the regrouping must take place before the problem can be done. This should remind participants of **Clear the Boards**.
   - Use base-10 blocks to help with the chart at the beginning. The pieces can be placed in columns. Example:
     - ▲ Place each number in the appropriate tens and ones columns according to place value.
     - ▲ Starting with the ones column, decide if subtraction is possible.
     - ▲ If so, subtract.
     - ▲ If not, regroup from the tens column by taking 1 ten (record the change in the adjusted tens column, leaving 3 for this example).
     - ▲ Add 10 ones to the ones column and record the new amount in the adjusted ones column (12 for this example).
     - ▲ The second number does not change. Just recopy that number (27).
     - ▲ Complete the subtraction.

Tens	Adjusted Tens	Adjusted Ones	Ones
4	3	12	2
2	2	7	7
	1	5	

   - A common error with this approach is that the learner may not have the skills to organize the chart.

3.4 Activity: Mail It

Paraeducators will use knowledge of place value to explore multiplication and division algorithms.

Materials:

- Base-10 blocks (or paper patterns)
- Counters
- **Mail It** handout and transparency **(H14/T19)**

3.4.1 Steps

- Use the **Mail It** handout and transparency **(H14/T19)**
- In pairs, solve the problem on the addition handout ***without the traditional algorithm.***
- Using the boxes provided on the handout, create the following mail order:

  Dear Mathematics Express: Please send 40 copies of your math book: "If It's About Numbers, Count Me In."

  Thank You, Mrs. Addison, 2nd-grade teacher

1. Show how many boxes would be needed to ship all of the books if each box holds 5 books. How many boxes are needed?
2. A late order of 4 more books was added. Show how many boxes are now needed to ship <u>all</u> of the books. How many boxes are needed?
3. Describe what your order looks like.
4. Explain how you got your answer.
5. What will the total cost be for the Mathematics Express to ship the boxes if every box costs $8? Explain how you got your answer.

- Once solved, relate work to the traditional algorithm steps for multiplication and division
- Discuss what parts of the algorithm are represented with each step when using the traditional algorithm of:

$$5\overline{)20}$$

- Use the following steps to guide the discussion:
  - ▲ This is division because we are taking a total and partitioning or dividing the amount.
  - ▲ I am looking for groups of 5 because 5 books were allowed per box.
  - ▲ I do not have a group of 5 in the tens column, so I can trade those in for ones, which gives me 40 ones – *checking to see if 5 goes into 4, which it does not, so we look at 40.*
  - ▲ Now I can separate into 8 groups of 5 – *5 times 8 is 40 so all of my books have been boxed.*
  - ▲ The extra 4 books do not fill a box but must be shipped, so I really need 9 boxes – *this is 5 into 44, but I already know about 40 so I have 4 left over (remainder) and need one more box to ship them.*
  - ▲ The total cost is 9 boxes x $8 = 72 – *this is 8+8+8+8+8+8+8+8+8 or 8 x 8 =64 + 8.*

3.5 Lecture: New Thoughts for Multiplication and Division

Paraeducators will be given a sample problem challenging them to first decide on the operation and then use strategies to figure out the answer.

There are again many algorithms for multiplication and division. Success will vary by learner. Use the transparency and handout **Algorithms: Multiplication and Division (T20/H15)** with the following problems for practice. These exercises should include the traditional algorithms for those who need a review.

Using the chalkboard, a wipe-off board, or the overhead projector, solve the problems with the class. Direct them to work on their handouts at the same time, recording the steps as you solve the problems. Review each of the following steps. Take your time and answer questions as you proceed. Check to be sure participants are following along with understanding.

Multiplication:

1. Repeated Addition

$$4 \bullet 12 = \quad \begin{array}{rcc} & 1 & 2 \\ & 1 & 2 \\ & 1 & 2 \\ + & 1 & 2 \\ \hline & 4 & 8 \end{array}$$

A review of repeated addition:

- May be written as two different repeated addition sentences (the above could have been written as 4 added 12 times).
- Link more complex problems to repeated addition (such as fraction addition).
- Common errors: Cannot break problem apart correctly.

2. Partial Products – 2 digit by 1 digit
   - Use the *distributive property concept* with expanded notation to break the problem apart and reassemble.
   - For example, 4 · 12 can be broken down for easier multiplication.
     We know that 12 = 10 + 2 in expanded notation. Multiplying by 4 implies that each place value is increased by a factor of 4 (distributive property).
   - See the chart below for place value.

Tens	Ones
1	2
x4	x4
4	8

3. Partial Products – 2 digit by 2 digit
   - Break numbers into expanded notation.

$$\begin{array}{rcc} & 3 & 7 \\ \text{x} & 5 & 4 \\ \hline \end{array} \rightarrow \begin{array}{rc} & 7 \\ \text{x} & 4 \\ \hline 2 & 8 \end{array} \rightarrow \begin{array}{ccc} & 3 & 0 \\ \text{x} & & 4 \\ \hline 1 & 2 & 0 \end{array} \rightarrow \begin{array}{ccc} & & 7 \\ \text{x} & 5 & 0 \\ \hline 3 & 5 & 0 \end{array} \rightarrow \begin{array}{cccc} & & 3 & 0 \\ \text{x} & & 5 & 0 \\ \hline 1 & 5 & 0 & 0 \\ + & 3 & 5 & 0 \\ & 1 & 2 & 0 \\ & & 2 & 8 \\ \hline 1 & 9 & 9 & 8 \end{array} =1998$$

Division

1. Repeated Subtraction
   - Link to multiplication process and subtraction.

$$
\begin{array}{r}
24 \\
-\ \ 4 \\ \hline
20 \\
-\ \ 4 \\ \hline
16 \\
-\ \ 4 \\ \hline
12 \\
-\ \ 4 \\ \hline
8 \\
-\ \ 4 \\ \hline
8 \\
-\ \ 4 \\ \hline
0
\end{array}
\qquad 4\overline{)24} = 6
$$

Goal 4: Enhance number sense by building estimation skills.

4.1 Activity: How Close Is Close?

Paraeducatars will use a variety of estimation strategies to develop number sense with basic operations.

4.1.1 Steps

- Divide the class into small groups.
- Complete the handout **How Close Is Close? (H16).**
- Direct the groups to look at the problems and then choose the best answer through ***estimation*** and explain their thinking. Do not solve the exact problem.
- After the groups have had time to complete the activity, use the transparency **How Close Is Close? (T21)** to review their answers. Ask participants from each group to review the thinking that individual members used to solve the problems.

How Close Is Close?

1. Estimate the following. Explain your thinking. Do not solve the exact problem.

   A. $32 + 76$ a) 100 b) 1000

   B. $\begin{array}{r} 34 \\ \underline{\times\ \ \ 7} \end{array}$ a) 21 b) 210

   C. $5\overline{)213}$ a) 400 b) 40 c) 4

2. Estimate the following. Decide whether your guess will be high or low. Explain your thinking. Do not solve the exact problem.

   a)
   $$\begin{array}{r} 6 \\ 3 \\ 2 \\ 5 \\ \underline{+\ 7} \end{array}$$

   b) 27 + 31 + 28

   c) $8\overline{)62}$

   d)
   $$\begin{array}{r} 1{,}219 \\ 5{,}912 \\ \underline{+\ \ 2{,}446} \end{array}$$

   e)
   $$\begin{array}{r} 568 \\ \underline{-\ \ 32} \end{array}$$

3. If you have $10, will you have enough for a sandwich costing $3.99, a dessert costing $1.09, and a drink costing $0.99?

4.2 Lecture: Number Sense and Estimation

Use **Number Sense and Estimation Strategies** handout and transparency **(H17/T22)** with this lecture after completing **Activity 4.1,** How Close Is Close?

Students need a sense of whether they are giving an appropriate answer. *Number sense* is the development of that skill. Number sense is the ability to determine if the final answer is possible or reasonable (use the transparency **Number Sense and Estimation Strategies [T22]**).

Estimation is a skill used to develop number sense. *Estimation* is the process of giving an approximate answer. Young students fear estimation because they worry about giving the wrong answer. Through computation, we give the impression that exact answers are the only important answers. Estimation is a valuable skill for predicting answers and validating solutions.

Estimation skills get stronger with practice. They gain in strength as students build a larger mathematical knowledge base. Number sense begins with understanding the operations as developed throughout this module. For young children, simply deciding if a number should get larger or smaller from an operation or guessing the size of a set without counting are all important activities for developing estimation skills.

Use the handout **Number Sense and Estimation Strategies (H17)** to talk through the following strategies. Participants will not understand each method by simply reading the handout.

Number Sense and Estimation

1a. Answer: A
- From place value, the answer must be in the hundreds.

1b. Answer: B
- From place value, we can look at the 7 and 3, noting that we already have 3 tens. Multiplying it by 7 would make it even larger.

1c. Answer: B
- 400 is not reasonable because we are dividing.
- From basic facts, we know that 5 x 4 = 20.
- B is the best answer as we look at dividing a hundreds number into groups.

High or low for the following will depend on the numbers used in the problem.

2a. Answer: Approximately 20 (slightly under)
- *Making 10s* – students quickly learn what numbers make 10 and that adding 10s and zeroes is simple.
- The answer can be over or under depending on the strategy.

2b. Approximately 90 (slightly high as two numbers were rounded up)
- *Clustering* – each number clusters around 30, so 30 x 3 = 90.
- Clustering only works if numbers are close to a single number.
- *Rounding* can also be used here.

2c. Approximately 8 (slightly high as the number was rounded up)
- *Compatible numbers* – changing a problem makes numbers easier to work with.
- We know from our facts that 8 goes into 64 evenly, so our answer will be slightly less than 8.
- Can also be linked with making 10s.

2d. Approximately 9000 (slightly high)
- *Front-end rounding* – for large numbers, front-end rounding can be useful.
- Without following rounding rules, a reasonable estimation may be found by looking only at the left-most digits.
- Rounding skills can make this more accurate as students learn rounding rules.

2e. Approximately 540 (slightly high)
- *Rounding* – simple rounding rules can be used here.
- We know that our number will not drop below 500 because only 30 is being taken away.
- Rounding rules generally say that we look to the right of the number we want to round to – if that number is 5 or greater we round the previous digit up; less than 5, we round down.
    - ▲ Rounding 568 to the nearest 10s rounds to 570 because 8 is greater than 5, so we increase 6.
    - ▲ Rounding 568 to the nearest 100s rounds to 600 because 6 is greater than 5, so we increase 5.

3. Answer: We will have plenty of money
    - We must round to the nearest dollar \$4 + \$1 + \$1 = \$6.
    - This is the most practical skill for estimation.

Goal 5: Define and communicate scenarios for appropriate use of basic operations (applications, money, time, etc.).

5.1 Lecture: That's What It's All About

Throughout instruction in basic facts and operations, students need the experience of applying these operations to actual applications. ***Some students will struggle with applications and not struggle with computation***. For example: Many learners can complete the computation "2x4=?" with ease and quickly give the answer "8." But when confronted with the opportunity to apply this computation to a real scenario such as "Mark buys two packs of baseball cards with four cards in each pack. How many cards does he have now?," these same learners often struggle and cannot apply the skill.

The vocabulary previously presented in lectures and activities should become a natural part of the paraeducators' math vocabulary. A list of associated words should also be kept in the math journal to help recognize what processes are required.

Computation is useless without something to compute; that's what problem-solving is all about. Successful problem-solvers have a "tool box" of strategies and basic skills from which to draw. With each operation, students should be encouraged to not only solve given problems that serve as models, but also to write problems of their own to demonstrate understanding. Expressing this need in words helps students make the link between their computation skills and using those skills to actually solve a problem. Another way to say this is that students demonstrate "application skills." Expressing the need in words also provides assessment information for both the student and the instructor.

The following activity asks participants to write application problems. The list of key words and phrases should be used somewhat naturally during this activity. Develop and review this list after the activity for any key words that participants have not used. This list may also be posted in the classroom for use throughout these modules. Use transparency and handout **The Language of Math Applications (T23/H18).**

The Language of Math Applications

The language of addition: *sum, total, all together, combine*
The language of subtraction: *difference, take away, gives*
The language of multiplication: *times, increases by*
The language of division: *divided by (into), shared equally*

The language of applications is based upon specific concepts:

- Addition and multiplication problems show the concept of "increase."
- Subtraction and division problems show the concept of "a decrease or partition."

Keep in mind: Correct vocabulary is used to imply operations.

5.2 Activity

Paraeducators will use prior skills and knowledge to write applications for grades K-4 for each operation.

5.2.1 Steps

- Have participants individually define what it means to add, subtract, multiply, and divide.
- Share ideas with small groups to revise the definitions.
- Make sure to include the effects on the operations on the answers (e.g., addition – the final answer is larger).
- Discuss processes.
- Discuss linked processes.
- Have each group of 4 students write an application problem (for any grade K-4) to demonstrate each operation.
- Trade problems with another group.
- Have groups solve problems using any method and supporting it with a written computational algorithm.
- Trade problems back for checking.
- Debrief as a group what key words were used in order to solve the problem (refer to the list above)
- List any words that were confusing in signaling the required operation.

Assignment Part 2 (of 2 parts)

****Note to Instructor:*** Decide how long the class has to complete the assignments so that you have time to grade assignments, record grades, and turn in materials from this course in a timely manner. If the paraeducators are taking the course for credit, there will be a time limit based upon the grading period at the attending institution. You also need to decide how you would like attendees to turn in their assignments. Options include mailing or whatever arrangements work for you and your class. **You are strongly encouraged to be firm about a completion date and may need to make some effort to follow up on attendees and their progress**. **A rubric for grading is provided**.

Hand out **Assignment Part 2: Subtraction Facts (H19)**. Read the instructions and answer questions regarding completion of the assignment. Provide the class with a date for completion and your process for handing the assignment in.

Assignment: Subtraction Facts

The following assignment is worth **125 total points.**

The focus of the assignment is to analyze subtraction facts using addition knowledge. Participants will use personal experience to answer the questions.
There are three steps to this assignment.

Step 1: (10 points)
The participant will analyze the addition facts chart used in earlier activities. The participant will explain how the chart works. Accurate completion of this step is worth a maximum of 10 points.

Step 2: (80 points)
The participant will use what he/she knows about the link between subtraction and addition to describe how the same chart may be used for subtraction. The participant will create rules to use with students explaining how to use the chart. Evidence of the inverse relationship of addition and subtraction and examples starting with the answer for each fact should be included. Accurate completion of this step is worth a maximum of 80 points.

Step 3: (35 points)
The participant will list two examples of difficulties students have with subtraction and share how the chart could help deal with those difficulties. Participants should make note of fact families (to link to addition), working with large numbers, etc. Accurate completion of this step is worth a maximum of 35 points.

Assignment 2 Handout: Subtraction Facts

Name:

Date:

The following assignment is worth 125 total points.

The focus of the assignment is to analyze subtraction facts using addition knowledge. Participants will use persoanl experiences to answer the questions.

There are three steps to this assignment.

Step 1: (10 points)
Analyze the attached addition facts chart used in earlier activities. Explain how the chart works for addition facts. Give examples that follow your explanation.

Step 2: (80 points)
Use what you know about the link between subtraction and addition to describe how the same chart may be used for subtraction facts. Create rules or steps that you could give to a student to explain how to use the chart to learn basic subtraction facts. Provide examples that show your rules at work.

Step 3: (35 points)
List two examples of difficulties that you have seen or experienced with students learning subtraction facts. Explain how you could use the chart to solve those issues. Provide clear examples of your solutions.

Module C
Handouts

Module Goals
Module C:
Number Representation and Manipulation

The paraeducator will:

1. Use multiple models to develop understandings of place value and the base-10 number system

2. Understand the meaning, effects, and relationships of the basic mathematical operations

3. Use patterns to explore algorithms for basic mathematical operations

4. Enhance number sense through building estimation skills

5. Define and communicate scenarios for the appropriate use of basic operations (applications, money, time, etc.)

Building the Foundation

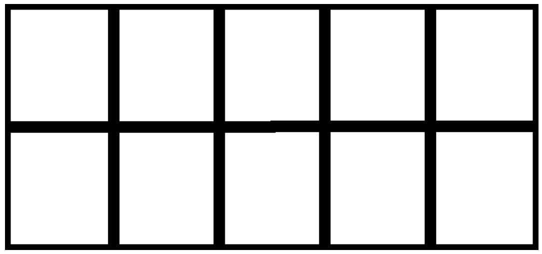
The foundation of all number concepts begins with ***place value***.

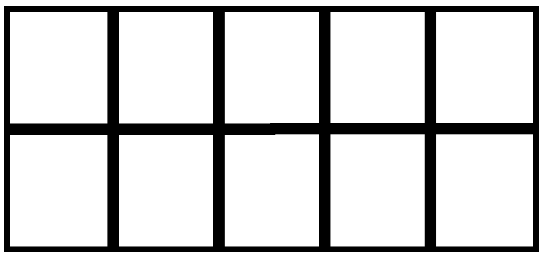
Place Value

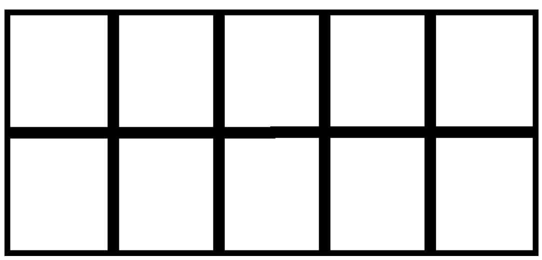
The value of a digit (how much it stands for) is based on where it is "placed" in a number sequence.

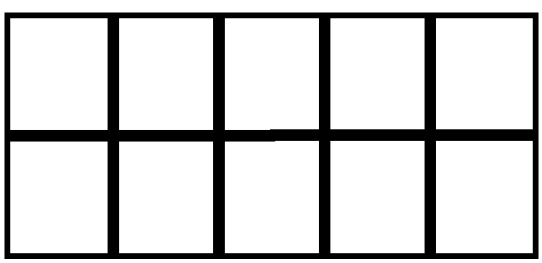
Our number system is based on a simple pattern of tens. It is called the *base-10 system*.

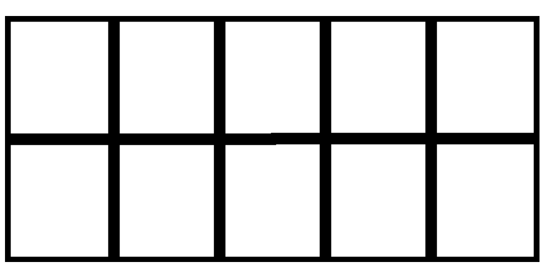
Ten Frame





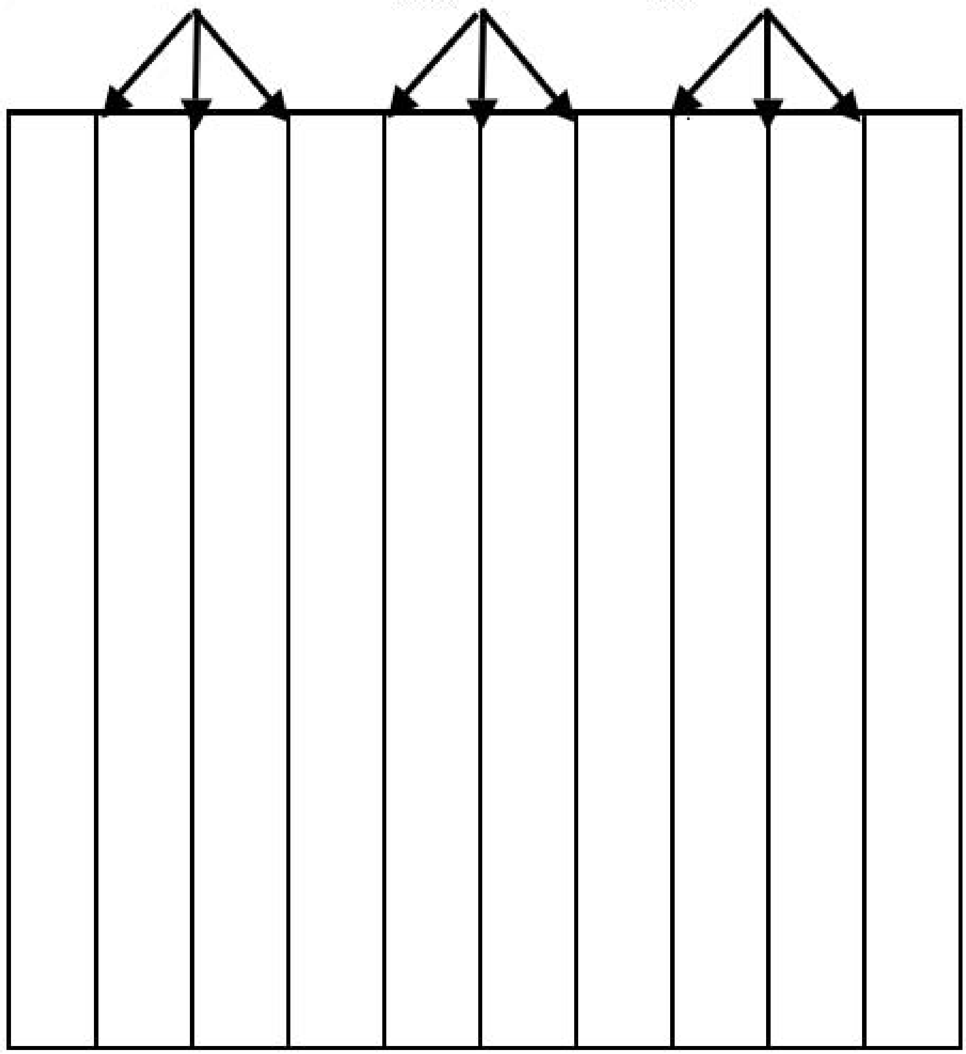






Paper Base–10 Blocks

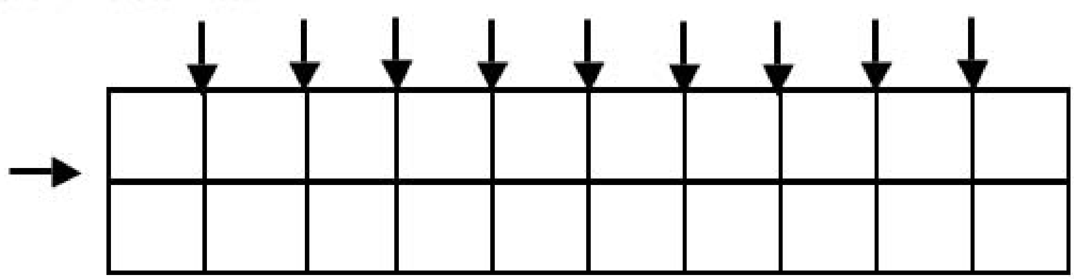
Longs: Cut Cut Cut



Units:

Cut on all cross lines:

Cut:



Flat

What's the Number?

Working in pairs, complete the following activity:

Using the numbers of the cards that the instructor drew from the deck:

1. Write the smallest number you can make with the cards. Show it with your blocks:

2. Write the largest number you can make with the cards. Show it with your blocks.

3. Using the blocks, in your pair model any number between the largest and smallest.

4. List the different numbers can you make with these three cards.

The Math Family

Addition is generally the first concept taught.

- Basic addition concepts come from counting.
- Mathematics in K-4 builds significantly on counting skills to master basic facts and arithmetic processes.
- Addition involves combining objects/numbers (*addends*) into a larger group (*sum*), thereby increasing the group size from either original group.
- The order of numbers does not matter for addition to produce the same answer.
- There are 100 basic addition facts.

Subtraction generally follows instruction in addition.

- Subtraction is the inverse (opposite) of addition.
- Subtraction starts with a larger group, and then some of the group is removed, producing the *difference* between the two numbers.
- The answer for subtraction is smaller than the starting number.
- Subtraction can be taught as counting backward or counting forward as in addition.
- There are 100 basic subtraction facts.

Multiplication is short-hand for addition.

- Every basic multiplication fact may be rewritten as addition. This is important as students learn basic facts.
- The order of numbers does not matter, which allows children to reorder numbers to utilize simpler counting patterns.
- Multiplication, like addition, combines two numbers (*factors*) and produces a larger number (*product*).
- There are 100 basic multiplication facts.

Division is generally the final basic concept.

- Just as addition and multiplication are related, so are subtraction and division.
- Division starts with a large number and is partitioned or divided into a specified number of groups, or groups containing a specified number.
- This is similar to subtraction in that the answer (*quotient)* is a smaller number than the beginning amount.
- Division is repeated subtraction.
- Division may also be taught in terms of multiplication facts.
- There are only 90 basic division facts because there are no division facts with zero as the *divisor* (number that you divide by).

Addition Facts

0	1	2	3	4	5	6	7	8	9
+ 0	+ 0	+ 0	+ 0	+ 0	+ 0	+ 0	+ 0	+ 0	+ 0
	1	2	3	4	5	6	7	8	9
	+ 1	+ 1	+ 1	+ 1	+ 1	+ 1	+ 1	+ 1	+ 1
		2	3	4					
		+ 2	+ 2	+ 2					
			3	4					
			+ 3	+ 3					
				4	5				
				+ 4	+4				
					5				
					+ 5				
						6			
						+ 6			
							7		
							+ 7		
								8	
								+ 8	
									9
									+ 9

Addition Facts
Diagonal

$\begin{array}{r} 0 \\ +\ 0 \\ \hline \end{array}$

$\begin{array}{r} 1 \\ +\ 1 \\ \hline \end{array}$

$\begin{array}{r} 2 \\ +\ 2 \\ \hline \end{array}$

$\begin{array}{r} 3 \\ +\ 3 \\ \hline \end{array}$

$\begin{array}{r} 4 \\ +\ 4 \\ \hline \end{array}$

$\begin{array}{r} 5 \\ +\ 5 \\ \hline \end{array}$

$\begin{array}{r} 6 \\ +\ 6 \\ \hline \end{array}$

$\begin{array}{r} 7 \\ +\ 7 \\ \hline \end{array}$

$\begin{array}{r} 8 \\ +\ 8 \\ \hline \end{array}$

$\begin{array}{r} 9 \\ +\ 9 \\ \hline \end{array}$

100 Addition Facts

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

100 Board

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	70
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Graph Paper

Clear the Boards

Materials:
- A pair of dice per pair of students
- Clear the Boards block mat
- Clear the Boards Record Sheet

Directions:
1. Each player needs a personal block mat.
2. Each player starts with a flat (100) in the hundreds column of the block mat.
3. The first player rolls the dice separately.
   - The first die represents numbers in the tens place
   - The second die represents numbers in the ones place
     Ex. First = 1, second = 7; the roll is 17
4. Take away the rolled amount from the hundred. This requires trading in so that the problem is possible.
5. Record the traditional algorithm on the record sheet and check to make sure it matches what is left on the board.

1	0	0
-	1	7
8	3	

6. Players take turns rolling, subtracting, and recording.
7. Once a player gets below 10, only 1 die will be used on the roll.
8. The first player to clear the board wins.

Block Mat

Hundreds	Tens	Ones

Clear the Boards
Record Sheet

Hundreds	Tens	Ones
1	0	0
–		
–		
–		
–		
–		
–		
–		
–		
–		
–		
–		
–		
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Hundreds	Tens	Ones
1	0	0
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Place Value Charts 1

Hundreds	Tens	Ones

Hundreds	Tens	Ones

Place Value Charts

Hundreds	Tens	Ones

Hundreds	Tens	Ones

Place Value Charts

Hundreds	Tens	Ones

Place Value Charts 2

Tens	Adjusted Tens	Adjusted Ones	Ones

Tens	Adjusted Tens	Adjusted Ones	Ones

Mail It

Use the boxes on the following handout to create the following mail order.

"Dear Mathematics Express:

Please send 40 copies of your math book "If It's About Numbers, Count Me In."

Thank you,

Mrs. Addison, 2nd-grade teacher"

1. Show how many boxes would be needed to ship all of the books. How many boxes are needed?

2. A late order of 4 more books was added. Show how many boxes are now needed. How many are needed?

3. Describe what your order looks like.

4. Explain how you got your answer.

5. What will the total cost be for Mathematics Express to ship the books if every box costs $8? Explain how you got your answer.

Matematics Express: A Math Mailing Service

Boxes for mailing math books: ***Each box can hold only 5 books.***

Algorithms: Multiplication and Division

Solve the following sample problems using the operations and strategies listed.

Multiplication

1. Use repeated addition to solve the problem:

   $4 \cdot 12 =$

   A review of repeated addition:
   - Can be written as two different repeated addition sentences. Link more complex problems to repeated addition (such as fraction addition).
   - Common errors: Cannot break problem apart correctly.

2. Partial Products – 2 digit by 1 digit
   - Use the *distributive property concept* with expanded notation to break the problem apart and reassemble the problem:

   $4 \cdot 12=$

2. Partial Products – 2 digit by 2 digit
   - Break numbers into expanded notation to show the answer to the multiplication problem:

$$\begin{array}{r} 3\ \ 7 \\ \underline{\times\ 5\ \ 4} \end{array}$$

Division

Repeated Subtraction
- Link to multiplication process and subtraction.

$$4\overline{)24}$$

How Close Is Close?

1. Estimate the following. Explain your thinking. Do not solve the exact problem.

   A. $32 + 76$ a) 100 b) 1000

   B. $\begin{array}{r} 34 \\ \underline{\times \quad 7} \end{array}$ a) 21 b) 210

   C. $5\overline{)213}$ a) 400 b) 40 c) 4

2. Estimate the following. Decide whether your guess will be high or low. Explain your thinking. Do not solve the exact problem.

   a) $\begin{array}{r} 6 \\ 3 \\ 2 \\ 5 \\ \underline{+\ 7} \end{array}$

   b) $27 + 31 + 28$

   c) $8\overline{)62}$

   d) $\begin{array}{r} 1{,}219 \\ 5{,}912 \\ \underline{+\ \ 2{,}446} \end{array}$

   e) $\begin{array}{r} 568 \\ \underline{-\ \ 32} \end{array}$

3. If you have \$10, will you have enough for a sandwich costing \$3.99, a dessert costing \$1.09, and a drink costing \$0.99?

Number Sense and Estimation Strategies

Number sense is the ability to determine if the final answer is possible or reasonable.

Estimation is the process of giving an approximate answer. Estimation is a skill used to develop number sense.

The strategies below are suggestions for solving the problem. A variety of strategies can be used.

Number Sense and Estimation

1a. Answer: A
- From place value, the answer must be in the hundreds.

1b. Answer: B
- From place value, we can look at the 7 and 3, noting that we already have 3 tens and then multiplying it by 7 would make it even larger.

1c. Answer: B
- 400 is not reasonable because we are dividing.
- From basic facts, we know that 5 x 4 = 20.
- B is the best answer as we look at dividing a hundreds number into groups.

2a. Answer: Approximately 20 (slightly under)
- *Making 10s* – students quickly learn what numbers make 10 and that adding 10s and zeroes is simple.
- The answer can be over or under depending on the strategy.

2b. Approximately 90 (slightly high as two numbers were rounded up)
- *Clustering* – each number clusters around 30, so 30 x 3 = 90.
- Clustering only works if numbers are close to a single number.
- *Rounding* can also be used here.

2c. Approximately 8 (slightly high as the number was rounded up)
- *Compatible numbers* – changing a problem makes numbers easier to work with.
- We know from our facts that 8 goes into 64 evenly, so our answer will be slightly less than 8.
- Can also be linked with making 10s.

2d. Approximately 9000 (slightly high)
- *Front-end rounding* – for large numbers, front-end rounding can be useful.
- Without following rounding rules, a reasonable estimation may be found by looking only at the left-most digits.
- Rounding skills can make this more accurate as students learn rounding rules.

2e. Approximately 540 (slightly high)
- *Rounding* – simple rounding rules can be used here.
- We know that our number will not drop below 500 because only 30 is being taken away.
- Rounding rules generally say that we look to the right of the number we want to round to – if that number is 5 or greater, we round the previous digit up; less than 5 we round down.
  - ▲ Rounding 568 to the nearest 10s rounds to 570 because 8 is greater than 5, so we increase 6.
  - ▲ Rounding 568 to the nearest 100s rounds to 600 because 6 is greater than 5, so we increase 5.

3. Answer: We will have plenty of money
- We must round to the nearest dollar \$4 + \$1 + \$1 = \$6.
- This is the most practical skill for estimation.

The Language of Math Applications

The language of addition: *sum, total, all together, combine*
The language of subtraction: *difference, take away, gives*
The language of multiplication: *times, increases by*
The language of division: *divided by (into), shared equally*

The language of applications is based upon specific concepts:

- Addition and multiplication problems show the concept of "increase."
- Subtraction and division problems show the concept of "a decrease or partition."

Keep in mind: Correct vocabulary is used to imply operations.

Assignment Part 2: Subtraction Facts

Name: ____

Date: _____________________________

The following assignment is worth **125 total points**.

The focus of the assignment is to analyze subtraction facts using addition knowledge. Participants will use personal experience to answer the questions.

There are three steps to this assignment.

Step 1: (10 points)
Analyze the attached addition facts chart used in earlier activities. Explain how the chart works for addition facts. Give examples that follow your explanation.

Step 2: (80 points)
Use what you know about the link between subtraction and addition to describe how the same chart can be used for subtraction facts. Create rules or steps that you could give to a student to explain how to use the chart to learn basic subtraction facts. Provide examples that show your rules at work.

Step 3: (35 points)
List two examples of difficulties that you have seen or experienced with students learning subtraction facts. Explain how you could use the chart to solve those issues. Provide clear examples of your solutions.

Assignment Part 2: Subtraction Facts

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Module C
Transparencies

Module Goals
Module C: Number Representation and Manipulation

The paraeducator will:

1. Use multiple models to develop understandings of place value and the base-10 number system

2. Understand the meaning, effects, and relationships of the basic mathematical operations

3. Use patterns to explore algorithms for basic mathematical operations

4. Enhance number sense through building estimation skills

5. Define and communicate scenarios for the appropriate use of basic operations (applications, money, time, etc.)

Building the Foundation

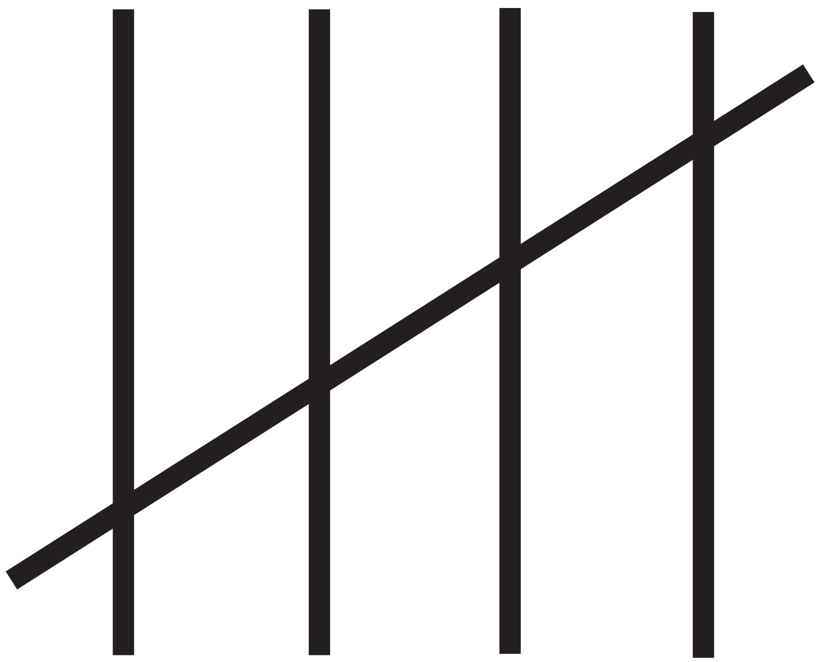
The foundation of all number concepts begins with ***place value***.

Place Value

The value of a digit (how much it stands for) is based on where it is "placed" in a number sequence.

Our number system is based on a simple pattern of tens. It is called the *base-10 system*.

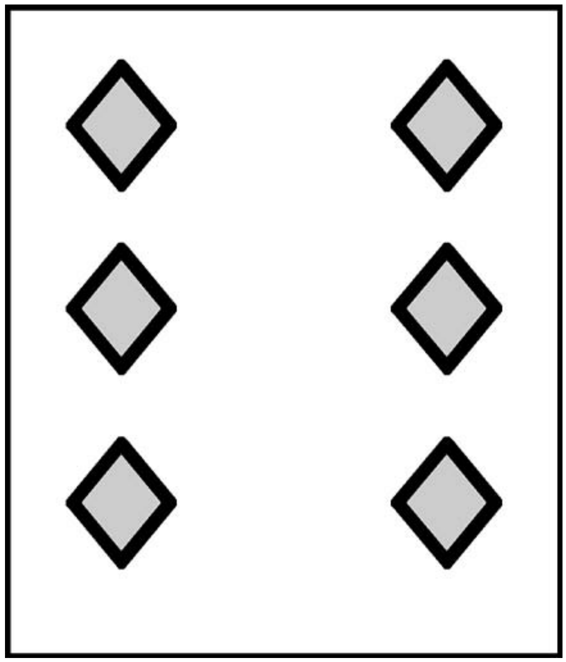
Tally

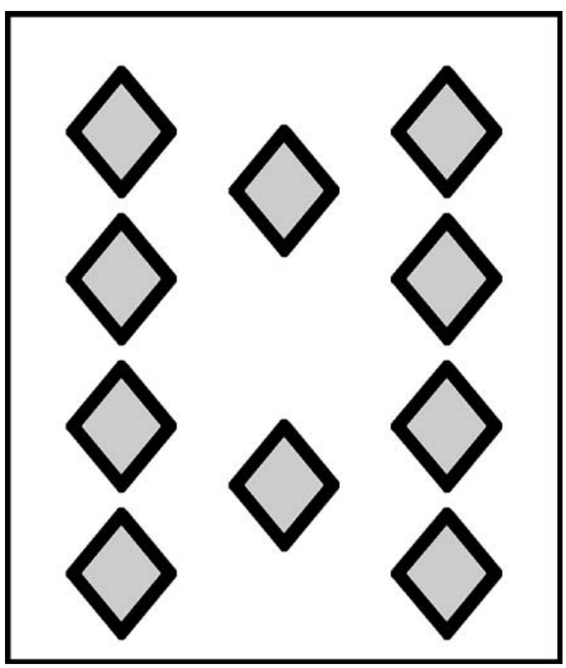


Grouping Data

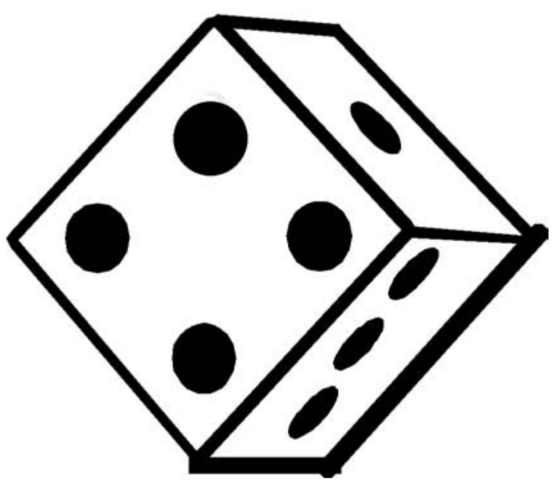
Grouping data makes counting easier and is one of the foundations of our number system.

Cards

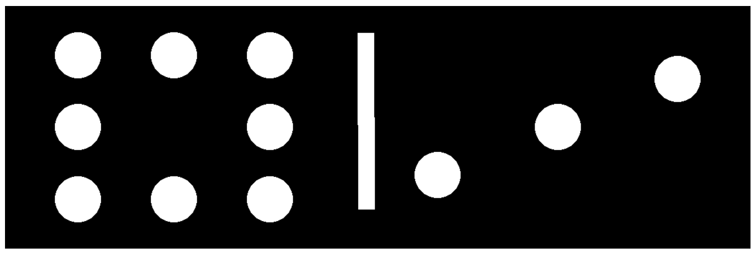


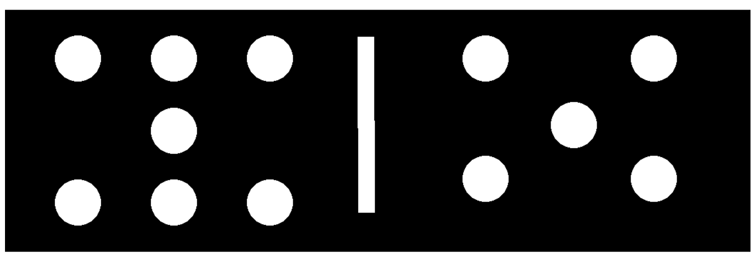


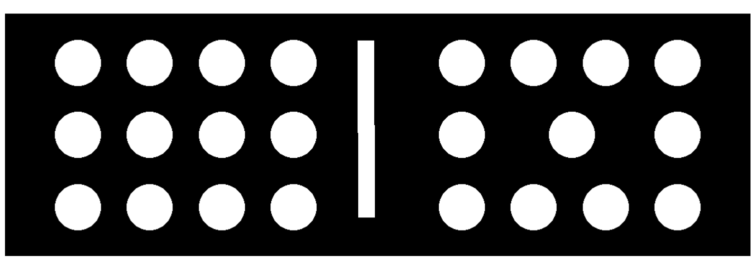
Dice



Dominoes 1







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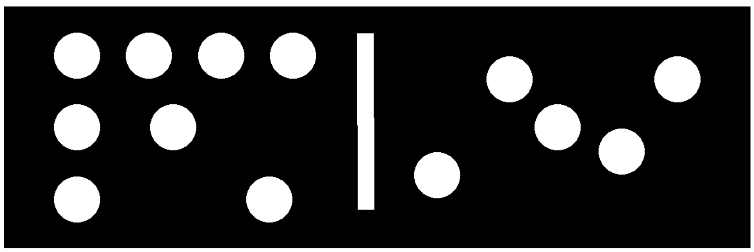
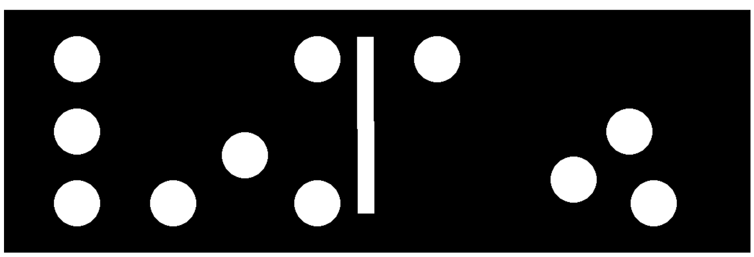
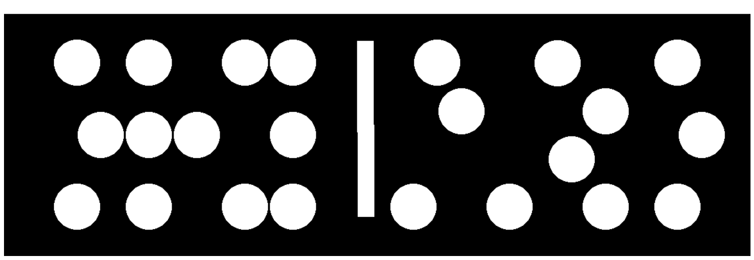






Ten Frame

The Math Family

- ***Addition***: Involves combining objects/numbers (***addends***) into a larger group (***sum***)

- ***Subtraction***: Starts with a larger group, and then some of the group is removed, producing the ***difference*** between the two numbers

- ***Multiplication:*** Combines two numbers (***factors***) and produces a larger number (***product***)

- ***Division***: Starts with a large number and is partitioned or divided into a specified number of groups, or groups containing a specified number. The answer (***quotient)*** is a smaller number than the beginning amount

100 Board

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	70
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Addition Facts

0 + 0	1 + 0	2 + 0	3 + 0	4 + 0	5 + 0	6 + 0	7 + 0	8 + 0	9 + 0
	1 + 1	2 + 1	3 + 1	4 + 1	5 + 1	6 + 1	7 + 1	8 + 1	9 + 1
		2 + 2	3 + 2	4 + 2					
			3 + 3	4 + 3					
				4 + 4	5 +4				
					5 + 5				
						6 + 6			
							7 + 7		
								8 + 8	
									9 + 9

Addition Facts

Diagonal

$$\begin{array}{r} 0 \\ +\ 0 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ +\ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +\ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ +\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ +\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ +\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ +\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ +\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ +\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ +\ 9 \\ \hline \end{array}$$

Patterns and Basic Facts

After you have colored the pattern of your choice on your basic facts handout, work with your group to look for:

- similarities and differences in patterns
- numbers that are colored on more than one board
- numbers that are seldom colored

Record all observations in your math journal.

Graph Paper

Clear the Boards

Hundreds	Tens	Ones

Place Value Charts 1

Hundreds	Tens	Ones

Hundreds	Tens	Ones

Place Value Charts 2

Tens	Adjusted Tens	Adjusted Ones	Ones

Tens	Adjusted Tens	Adjusted Ones	Ones

Mail It

Mathematics Express: A Math Mailing Service

Order placed: 40 copies of math book

- How many boxes?
- 4 books in late order
- What does the order look like?
- Explain how you got your answer.
- What is the total cost if they ship at $8 per box?
- Explain how you got your answer.

Algorithms: Multiplication and Division

Multiplication:

1. Repeated Addition

$4 \cdot 12 =$

2. Partial Product: 2 digit by 1 digit

$4 \cdot 12 =$

Tens	Ones

Algorithms: Multiplication and Division

3. Partial Products – 2 digit by 2 digit

$$\begin{array}{r} 3\ \ 7 \\ \underline{\times\ 5\ \ 4} \end{array}$$

Division:

Repeated Subtraction

$$4\overline{)24}$$

How Close Is Close?

1. Estimate and do not solve the exact problem.

A. $32 + 76$ a) 100   b) 1000

B. $\begin{array}{r} 34 \\ \underline{\times \quad 7} \end{array}$ a) 21   b) 210

C. $5\overline{)213}$ a) 400   b) 40   c) 4

2. Estimate the following. Decide whether your guess will be high or low. Explain your thinking. Do not solve the exact problem.

a) $\begin{array}{r} 6 \\ 3 \\ 2 \\ 5 \\ \underline{+\ 7} \end{array}$ b) $27 + 31 + 28$ c) $8\overline{)62}$

d) $\begin{array}{r} 1{,}219 \\ 5{,}912 \\ 5{,}912 \\ \underline{+\ \ 2{,}446} \end{array}$ e) $\begin{array}{r} 568 \\ \underline{-\ \ 32} \end{array}$

3. If you have \$10, will you have enough for a sandwich costing \$3.99, a dessert costing \$1.09, and a drink costing \$0.99?

Number Sense and Estimation Strategies

Number sense is the ability to determine if the final answer is possible or reasonable.

Estimation is the process of giving an approximate answer. Estimation is a skill used to develop number sense.

The Language of Math Applications

The language of addition: *sum, total, all together, combine*

The language of subtraction: *difference, take away, gives*

The language of multiplication: *times, increases by*

The language of division: *divided by (into), shared equally*

The language of applications is based upon specific concepts:

- Addition and multiplication problems show the concept of "increase."
- Subtraction and division problems show the concept of "a decrease or partition."

Keep in mind: Correct vocabulary is used to imply operations

Module D
Instructor's Guide

Module D:
Equivalency and Number Comparison

A. Lecture: Module Goals

Use the **Module D Goals** transparency and handout **(T1/H1)** to review module goals.

Module D: Equivalency and Number Comparison
The paraeducator will:

- Use a variety of concrete materials to develop meanings for commonly used fractions and decimals (e.g., 1/4, 0.5, 1/3 for sets and wholes
- Demonstrate equivalent forms of the same number through use of models, drawings, or other strategies
- Compare numbers (equal, greater than, less than) using a variety of strategies

Goal 1: Use a variety of concrete materials to develop meanings for commonly used fractions and decimals (e.g., 1/4, 0.5, 1/3) for sets and wholes.

1.1 Lecture: Fractions and Decimals

Remind participants to use their notebook to record important and critical definitions and notes.

In many texts, fractions and decimals are taught separately. Commonly, students struggle a great deal more with fractions than with decimals. **In reality, fractions and decimals represent the same concept. They are two different notations for the same value**. This concept will be developed throughout this module.

For grades K-4, familiarity with common fractions and decimals must be developed for both sets and wholes. Use the **A Fraction** transparency **(T2)**, explain that a *fraction* is the representation of part to whole. The whole can be one single object divided into parts or part of a set of objects. Learners need practice with both concepts. An example of "a single object divided into parts" would be a pie that is being served to 8 people. One single pie would be divided equally among 8. An example of "a part of a set of objects" would be a box of Legos containing 48 pieces being divided equally among 4 children.

1.2 Activity: Practice with Pieces

Paraeducators will participate in a short activity devising examples of sets and wholes.

1.2.1 Steps

- Ask participants to work in pairs.
- Use the transparency and handout **Partitioning Sets and a Whole (T3/H2).**
- Direct participants to discuss and determine three examples of each of the terms, such as the ones used previously in the lecture of the pie and the legos.
- Ask pairs of participants to share their examples with the class and record their examples of the transparency.

1.3 Continued Lecture: Fractions and Decimals

Decimal notation is a related notation. The decimal system relates to our base-10 system. Understanding of the base-10 system has been developed throughout this Academy. Numbers to the right of the decimal point represent a part of a whole; the whole is any number(s) to the left of the decimal point. The numbers to the right of the decimal and fractions are equivalent concepts. Fractions and base-10 concepts are necessary as conversions are often required to represent complex decimals.

We will begin with fraction concepts for visualization and then link these concepts and visualization to the use of decimals.

1.4 Activity: Break It Up

Paraeducators will develop the concept of common fractions by partitioning a variety of objects.

Materials:
- 4 different-colored strips of construction paper (at least 1”x6”), one of each color per class member

1.4.1 Steps

- Explain that for the purposes of this activity each strip of paper represents a *whole* strip of taffy – make note that all original pieces of taffy are equal in length but are different colors or “flavors.”
- Begin with one color and explain that you would like each participant to share that piece equally with a friend.
  - ▲ Ask them, how many pieces would they need? (2)
  - ▲ Should they be exactly the same size? (yes – they must be exactly the same size to be equal)
- Direct participants to fold the taffy to make two equal pieces (do not tear in half).
- Use other flavors of taffy to create 3, 4, and 8 equal pieces .
- Re-emphasize that pieces must be *equal.*

- Return to the piece with 2 equal parts.
- Link this piece to the word *halve,* which implies 2 equal parts. Write this on the chalkboard or transparency. Encourage class members to record the same information in their notes. Do the same with:
  - ▲ 3 = *thirds*
  - ▲ 4 = *fourths*
  - ▲ 8 = *eighths*
- Ask how many equal parts would be needed for fifths, sixths, etc.
  - ▲ *Fifths* = 5 equal parts
  - ▲ *Sixths* = 6 equal parts
- Return to the example of halves.
- Explain that you are going to take 1 piece.
- Ask, "what part of the whole did I take?"
  - ▲ You took 1 part of 2 or 1/2 = $\frac{\text{(part)}}{\text{(whole)}}$
  - ▲ Write the above example on the chalkboard or transparency. Review that the number on the bottom in a written fraction always represents the whole, and the number on the top always represents the part or chosen pieces.
  - ▲ Each piece represents 1/2; label each piece as 1/2
  - ▲ Note that 2 halves make a whole.
- Repeat the same activity for thirds, fourths, eighths.
  - ▲ *Thirds*: each piece is 1 of 3, so each is 1/3
  - ▲ *Fourths*: each piece is 1 of 4, so each is 1/4
  - ▲ *Eighths*: each piece is 1 of 8, so each is 1/8
- Review an important fact: The *denominator* (bottom number) represents the whole **AND** also tells how many equal pieces are demonstrated in the whole.
- Review the important fact: The *numerator* (top number) tells how many pieces have been chosen.
- Return to the strips, use the *fourths* strip. Explain that you would like to demonstrate 2 pieces of the *fourth* strip and can show this as:
  - ▲ 2 out of 4 pieces, so 2/4
- Use the handout **Fraction Pictures (H3)** for practice. Direct participants to individually:
  - ▲ Shade 1/2 of the circle.

    Note to Instructor: Participants should divide the circle into two equal pieces and shade one.
  - ▲ Shade 2/4 of the circle.

    Note to Instructor: Participants should divide the circle into four equal pieces and shade two.
  - ▲ Shade 1/4 of the square.

    Note to Instructor: Participants can divide the square into four equal parts in several ways – share the multiple ways that the class creates with the entire class.
  - ▲ Shade 2/3 of the rectangle.

    Note to Instructor: The participants should divide the rectangle into three equal parts from a possible two ways and shade two.

- Direct participants to move around the room after completing the activity, comparing their responses to those of other participants.
- Note that each fraction can look differently depending on the shape, but the relationship is the same as long as the whole is defined.
- Using shapes other than rectangles is important, ***otherwise students begin to associate fractions with rectangles rather than understanding the concept of any whole.***
- Use the transparency **Fraction Pictures (T4)** to review the process and clear up any confusion or disagreement in the drawings observed.

1.5 Discussion: Fractions as Sets

Picturing fractions as "partitioning a whole" is usually easier than picturing fractions as parts of a set of objects.

Pose the following problem:

"I have 15 marbles. Partition this set of marbles into 5 equal groups (parts)."

Allow groups to use whatever materials they find necessary to come up with the answer. The response to the problem should be:

In each of 5 equal groups, there should be 3 marbles.

Ask the group what concept this reminds them of. They should link this to division.

Fractions are short-hand for division notation explored in grades 5-8. For grades K-4, realizing that a set can be divided is equally important.

Continue the discussion. Ask the following question:

- "Each group equals what fractional part of the set?" The answer is 1/5.

A common incorrect answer is "3." Students count the objects in the smaller groups rather than looking at the total number of groups in the set. This is sometimes difficult for students. Revisit the concept of the whole. While the whole set was 15 marbles, the question asked us to divide it into 5 equal groups (of 3 per group), which becomes the "whole" concept. To help clarify this concept, use the following example. Take your time and make sure that class members have access to manipulatives to help them more accurately visualize the concept.

Pose the following three questions using the marbles from above to ensure clarity. Reading what is asked is extremely important. Question 3 is typically used in grades 5-8 for multiplication of fractions. Young children need to focus on the concept of "wholes and naming sets."

1. Two parts are how many groups of the whole marble set? (2 groups)
2. List that fraction. (2/5)
3. How many marbles are in 2 groups (parts)? (6 marbles). So, 2/5 is the same as 6 marbles out of 15.

1.6 Discussion: Decimals

Explain that, like fractions, decimals can also be used to represents parts of a whole. Decimals follow the same concept as our regular base-10 systems in that each place value is a grouping of 10. Base-10 blocks can be used in the same manner as for whole numbers, except the pieces must be redefined. Use the transparency **Decimal Place Value Chart (T5).**

Decimal Place Value Chart

Place value concepts for whole numbers. Example:
1 hundred + 2 tens + 5 ones = 125

Hundreds	Tens	Ones
1	2	5

A decimal point is used to mark the division between wholes and parts. Numbers to the right of the decimal point are parts of a whole. Those to the left of the decimal point are whole numbers. New vocabulary is necessary for naming decimal places. Encourage participants to record in their notes the new vocabulary necessary for naming decimal places.

Note: There is no "oneths" place to the right of the decimal point – the decimal is the boundary between wholes and parts

Hundreds	Tens	Ones	.	Tenths	Hundredths
1	2	5		3	7

This number is said "one hundred twenty-five *and* thirty-seven hundredth." The decimal is the word *and* signifies the break between whole and part.

For decimals, we are *only working with groups of 10*. Relate back to fractions using the handout **Decimals (H4)** and complete the following practice activities.

Decimals

- **#1: a)** Divide the blank square into tenths (10 parts with 9 lines).
  - Note that tenths imply 1 whole being divided into 10 parts.

  b) Shade 3 tenths (3 strips).

  c) Write the above as a decimal.

  Note to Instructor: To write 3/10 as a decimal, we need a decimal point to show less than 1 whole. The tenths place is the first place past the decimal, so we place a 3 giving 0.3. The zero is used to show no wholes.

- **#2: a)** Using the flat on the handout, shade in 25/100 (25 squares).
  Note to Instructor: This time the whole, although using the same square as in #1, is divided into 100 parts. Each square is 1/100.
  b) Write the above as a decimal.
  Note to Instructor:The hundredths place is 2 places past the decimal because the parts are divided into even smaller pieces. We again need a decimal to show less than a whole. How might we write 25 out of 100 as decimal? (0.25).

It is useful to put blank lines up to the smallest place value needed.

0. __ __

This time, we have 2 blanks to fill in, so 25 is placed after the decimal point and is read "twenty-five hundredths."

This should be related back to the place value chart as 2 tenths and 5 hundredths due to place value. Draw a place value chart on the board or on the transparency and review the placement.

0.	Tenths	Hundredths
	2	5

With place value blocks, the *long is now a tenth*. The *ones are now hundredths*. Show this to the group.

Ask the class: "What is the *flat* in decimal notation for this activity?" (1 whole)

Please note and review with the class that for grades 5-8 the blocks can be reassigned values as the place value moves further out to thousandths (whole = 10x10 cube, flat = tenths, long = hundredths, ones = thousandths).

As a class complete the handout. Allow time for thinking and questioning. Ask class members to complete each problem independently and share their reasoning with a neighbor. Then review the problem as a group using the answers and instructor notes provided.

- **#3:** Shade 5 hundredths. Write as a decimal.
  ***Note to Instructor**: There are no tenths, so a zero must be placed in the tenths place to fill in the space; the decimal is 0.05, and the model shows 5 units (hundredths).

- **#4**: Shade 10 tenths. Write as a decimal and draw the model.
  ***Note to Instructor:** The decimal is 1.0. and a flat is used to show a whole. The entire flat should be shaded.
  - ▲ This should generate discussion about what happened. Make sure that you review:
    1. As with regular place value, once 10 is reached, we must regroup.

2. The tenths only allowed one place value, so we had to move to a larger place value, that is, to the whole.
3. 1.0 shows 1 whole or 10 tenths.

Discuss possible pictures for the same concept: either 10 tenths or 10 columns of the hundredths picture – either way shade a whole.

- **#5:** Shade 10 hundredths. Write as a decimal.
  ******Note to Instructor***:** Shade 10 squares on the hundredths picture requiring 2 place values; the decimal is 0.10, with 10 hundredths (units) traded up for 1 long as denoted by the place value.

- **#6:** Shade 100 hundredths. Write as a decimal.
  ******Note to Instructor***:** Shade 100 squares on the hundredths. Shade all of the squares. The decimal is again 1.00 or 1.0
  - ▲ Students should see a link to #4 as 100 hundredths cannot be contained in 2 place values, so we must trade up for 1 whole.
  - ▲ Discuss possible pictures for the same concept: either 100 hundredths or 10 tenths – either way shade a whole.

While the class is completing the discussion and activity, make note with them of the difficulties they are encountering. Most do well coloring in the required pieces due to their knowledge of fractions but have difficulty modeling with the new definitions for the base-10 blocks.

The concept of 1 whole is foundational to mathematics all the way through high school. Recognizing different forms of 1 whole is an important developmental step that will take much practice.

Goal 2: Demonstrate equivalent forms of the same number through use of models, drawings, or other strategies.

2.1 Activity: Makin' Change

Paraeducators will use money values to develop equivalent forms.

Materials:

- Real or paper coins
- 1 set of dice or playing cards (discard face cards and joker)
- Handout **Makin' Change (H5)**

2.1.1 Steps

- Divide class members into pairs.
- Explain that money is an easy place to start when developing concepts of equivalent forms.

- Very young students should focus on coin recognition and the value of the coin system.
- Students in K-4 need to be able to show money values in different forms (e.g., $0.52 = 2 quarters + 2 pennies or 5 dimes + 2 pennies).
- Money can be difficult because it is a *non-proportional* system (regrouping occurs at different values) unlike base-10, which is *proportional* (each place value is 10).
- To begin, roll the dice or draw two cards to create a cent value.
- Have each pair create the value in as many ways as possible on the handout **Makin' Change**. Using the number chart on the handout may help keep the data organized.

Quarters	Dimes	Nickels	Pennies

- Have players should record and communicate their equivalent forms to show different combinations:
  - ▲ 5 pennies = 1 nickel
  - ▲ 23 cents = 4 nickels + 3 pennies or 2 dimes + 3 pennies
- For each value, pairs should share their number of combinations and demonstrate any that the group questions (the instructor can replicate on the overhead as necessary).
- Repeat as appropriate.
- Class members must realize that some combinations are easier than others, especially if they are carried in a pocket (25 pennies vs. 1 quarter).
- Discuss the grouping issues and when regrouping occurs with different coin values. Example: 5 nickels can be regrouped as equivalent form of 1 quarter, or 4 quarters can be regrouped as an equivalent form of $1, etc.

2.2 Activity: Change for a Dollar

Paraeducators will use equivalent forms and subtraction knowledge to develop money skills.

Materials:

- Real or paper coins
- 1 set of dice or playing cards (discard face cards and joker)

2.2.1 Steps

- Continue to work in pairs or in small groups.
- Roll the dice or choose two cards for a starting number of "cents" (e.g., you chose or rolled a 5 and a 2).
- Direct participants to create the value with the *least* number of coins possible:
  - ▲ 52¢ = 2 quarters and 2 pennies
- Explain that a customer purchased an item that was 52¢ and only had $1.00.

- The cash register was broken, so the clerk had to count back change.
- Explain that when counting back, the most effective way is to get to some easy number to count from instead of counting back all of the change in pennies.
- In the current example, one way would be to start at 52¢, and count three pennies → 53, 54, 55.
- It is then easy to add a nickel to 55, counting by five, → 60.
- Next, additional dimes may be added, counting by tens, until we reach $1.00 → 70, 80, 90, 100.
- This is a long way to make change but it works well for an introduction to this concept.
- Other methods include counting with pennies to 55, and then adding/counting with 2 dimes (65, 75) and then 1 quarter (100).
- This skill involves ***skip counting*** as counting nickels counts by fives and counting dimes counts by tens; pennies only count by ones.
- This skill takes a great deal of practice.
- Repeat the steps above making change to $1 from the following amounts: 41¢, 73¢, 11¢, 18¢.

2.3 Activity: Fractions = Decimals

Paraeducators will use prior knowledge to create equivalent forms of the same number.

Materials:

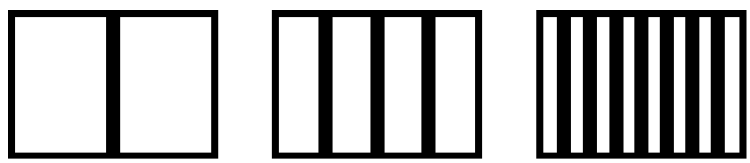
- “Taffy strips” (fraction strips) of 4 colors of paper, pre-folded from activities already completed
- Transparency and handout **Decimal Paper (T6/H6)**

2.3.1 Steps

- Continue working in pairs or small groups.
- Ask participants to use the strips of paper used for the fraction activities presented earlier (the 4 colored strips called “taffy”).
- Ask the participants to examine their paper strips and note that the strips have been folded for the previous activity. Explain that in the current activity they will be using the same fold lines and **will not** create new folds in the paper strips.
- Explain that now that they have the concept of equivalent forms in their minds from the money exercise, it is appropriate to move on to more abstract concepts.
- Stress that linking fractions and decimals through multiple models is a vital connection for developing math concepts.
- Using the fraction strips from the earlier activity, begin by left aligning the 4 strips so that all are visible.
- Point out that all strips are the same length, implying that they are all ***equivalent***.

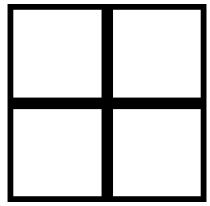
- Locate the halves strip, then fold under 1/2 so that only 1/2 is showing.
- Working in pairs or small groups, place the half strip on top of the other strip, looking at it carefully to make sure that it aligns with a previous fold line of thirds, fourths, and eighths. Do not make new folds or creases.
- Record which ***fractions*** are equivalent. Check for the following results:
  - ▲ *Fourths* – 2/4 were the same length as 1/2
  - ▲ *Thirds* – not possible to get equivalent length
  - ▲ *Eighths* – 4/8 were the same length as 1/2
- Summarize that 1/2 = 2/4 = 4/8 (look for patterns)
- Discuss what happened with thirds (three is not a multiple of 2 but can get 4 and 8)
- Predict what other numbers might work for equivalent to 1/2 (5/10 = 6/12 = 7/14 = 8/16, etc.)
- Using 3 identical black squares templates, partition and shade each square to represent , paying attention to the organization of the divisions; the goal is to show equivalence. Use transparency **Equivalent Fractions (T7).**

  One partitioning example is shown below:



- Note that the *area* colored on all three squares is equal but the partitions are different. This proves that our fractions are *equivalent.*
- As we can prove equivalent fractions, we must only link to equivalent decimals.
- Using a tenths template on the handout **Decimal Paper (H6),** each person should shade 5 tenths.
  - ▲ List the decimal (0.5)
  - ▲ List the fraction (5/10)
- Using another tenths template on the handout, direct each person to shade 1/2 of the strips.
- List the fraction shaded (1/2) (reduced fraction form).
- Compare the 2 templates (they are equal). Show on transparency **Decimal Paper (T6)**.
- Therefore, 1/2 = 5/10 = 0.5 shows that these are *equivalent forms.*
- Assign the following problems for classroom/group work:
  Problem: Shade 25 hundredths (choose the appropriate template – the hundredths flat)
  - ▲ List the decimal (0.25)
- An equivalent fraction is needed for this decimal.
  - ▲ Use fraction of shaded hundredths to total hundredths (hundredths template) and the result is 25/100.
- The fraction should be reduced to its lowest terms (or reduced form) in order to recognize the common fraction equivalent.
  - ▲ On a blank hundredths template, ask each person to partition the 100

squares into equal groups by drawing outlines, for example:



- ▲ Encourage participants to refer to previous shadings for fractions.
- ▲ Record the results on the overhead for the following: 2 groups of 50, 50 groups of 2, 5 groups of 20, 20 groups of 5, 4 groups of 25, and 25 groups of 4.
- ▲ For their chosen partitioning, each person should again shade 25 squares by carefully filling in an entire group before moving to the next group.
- ▲ Ask the following questions. Use transparency **Decimal Paper (T6)** to clarify:
- ▲ Which groupings worked for 25 by having no incomplete groups? (20 groups of 5, 4 groups of 25)
- ▲ Which groupings did not work for 25 by having an incomplete group? (all the rest listed above)
- ▲ Each participant should replicate these two groupings on their handout and name each shaded fraction that worked, 25/100 = 5/20 = 1/4.

  ✒***Note to Instructor:*** Make sure that fractions are representing shaded groups vs. unshaded groups, not in individual squares.
- ▲ The common fraction in its lowest terms is the simplest fraction or the one with fewest partitions is 1/4.
- ▲ Therefore, 25/100 = 0.25 = 1/4.
- ▲ Coins may also be used (25 cents is represented by 25 out of 100 pennies or 1 of 4 quarters).

  ✒***Note to Instructor:*** Shorter forms of reducing fractions are introduced in grades 5-8. The focus for the K-4 curriculum should only be on the use of simple common fractions that are easily reduced.

- Try an additional example for common fractions:
  - ▲ Shade 75 hundredths on a template.
  - ▲ List the decimal (0.75).
  - ▲ Name the equivalent fraction (75/100).
  - ▲ Review partitions that work: 20 groups of 5 and 4 groups of 25.
  - ▲ Equivalent fractions: 75/100 = 15/20 = 3/4.

Goal 3: Compare numbers (equal, greater than, less than) using a variety of strategies.

3.1 Lecture: Comparing Numbers

Comparing numbers is a difficult process for many students, especially when comparing fractional values.

Throughout the work with fractions, it is hoped that participants has developed some recognition of the partition size compared to the actual denominator. If not, now is the time to reiterate that observation. Look again at the fraction strips and have participants fold back all but one partition of each strip. Order them from smallest to largest piece. Discuss the relationship of the denominator to the partition size. Note that the larger the denominator, the smaller the piece. This can be related to sharing a pizza with three people, four people, and eight people. The more people, the smaller piece each gets.

We have already explored equality. Equality is not the only important relationship. As children increase their mathematical knowledge, we proceed to comparing numbers for size. It is appropriate to move from deciding that one piece is larger than another to using the symbol > (greater than) to show that one is larger than another (or < to show less than or smaller).

Regarding the fraction strips that were used previously, we can say that 1/8 is smaller than 1/3 or 1/8<1/3. For numerators other than 1, we must be careful in our decision about greater or less than depending upon the size and amount of the pieces.

3.2 Activity: Comparison Conundrum

Paraeducators will use fraction and decimal knowledge to order uncommon fractions and decimals.

Materials:

- Fraction strips from prior activities
- Additional construction paper to make strips or handout **Fraction Bars (H7)**
- Transparency and handout **Comparison Conundrum (T8/H8)**

3.2.1 Steps

- Ask participants to continue to work in pairs but to work with someone whom they have not worked with in the last few activities.
- Begin with simple whole number comparisons using a number line starting at zero. Use transparency and handout **Number Line (T9/H9)**.
- Important concepts:
  - ▲ Numbers get larger as they move right on the number line.
  - ▲ The place value farthest to the left begins the comparison.
- Introduce **> (greater than)** and **< (less than)** symbols.
  - ▲ The comparison sentence reads left to right just as though reading a sentence of written English.
    - o reads "Three is less than 5," which is true.
  - ▲ A helpful tool for remembering the comparison symbols is "The alligator (symbol) opens to eat the largest number."

- Point out the number line on the handout – make sure everyone knows how to read a number line and is able to locate the numbers on it. *Note: due to limited space on the handout, the smaller number line is not complete.*
- Groups can complete the following on handout **Comparison Conundrum,** #1-#3. Using the **Number Line** handout, insert the symbol and write the matching sentence. Write the first portion of the problem on the chalkboard or transparency, and then ask the working groups or pairs to complete the sentence: (e.g., 8___9).
  - ▲ #1: 8 __ 9 → < , 8 is less than 9.
  - ▲ #2: 11 ___ 21 → <, 11 is less than 21.
  - ▲ #3: 101 ____ 100 → >, 101 is greater than 100.
- After the group has worked with whole numbers, they can begin to work with fractions and decimals.
- Fractions require looking at:
  - ▲ the denominator as a reference, and
  - ▲ the numerator value
- Groups or pairs should be assigned the following, #4-#7, before an explanation is given – **encourage them to use all available tools to compare the fractions using the symbol and sentence.** Write the first portion of the problem on the chalkboard or the transparency just as #1-3 above.
  - ▲ #4: 1/3 _______ 2/3 →
    Answer: <, One third is less than two thirds (the pieces are the same size because the denominator is the same; must count the pieces determined by the numerator).
  - ▲ #5: 1/4 _______ 1/5 →
    Answer: >, One fourth is greater than one fifth (fifths are smaller than fourths).
  - ▲ #6: 2/3 _______ 2/8 →
    Answer: >, Two thirds is greater than two eighths (this time both denominator and numerator must be considered; use the fraction strips).
  - ▲ #7: 1/2 _______ 6/12 →
    Answer: =, One half is equal to six twelfths.
- Check the answers with groups and ask them to share the processes they used to determine their answers, such as they:
  - ▲ Colored same-area objects
  - ▲ Used fraction strips
  - ▲ Visualized
- Assign groups to complete #8-11 for decimals before giving an explanation. **Encourage them to use all available tools to compare the decimals using the symbol and sentence.**
  - ▲ #8: 0.1 ____ 0.2 →
    Answer: <, one tenth is less than two tenths.
  - ▲ #9: 0.26 ____ 0.25 →
    Answer: >, twenty-six hundredths is greater than twenty-five hundredths.

Module D
Handouts

Module Goals

Module D: Equivalency and Number Comparison

The paraeducator will:

1. Use a variety of concrete materials to develop meanings for commonly used fractions and decimals (e.g., 1/4, 0.5, 1/3) for sets and wholes

2. Demonstrate equivalent forms of the same number through use of models, drawings, or other strategies

3. Compare numbers (equal, greater than, less than) using a variety of strategies

Partitioning Sets and a Whole

A single object (whole) divided:

1.

2.

3.

Part of a set divided:

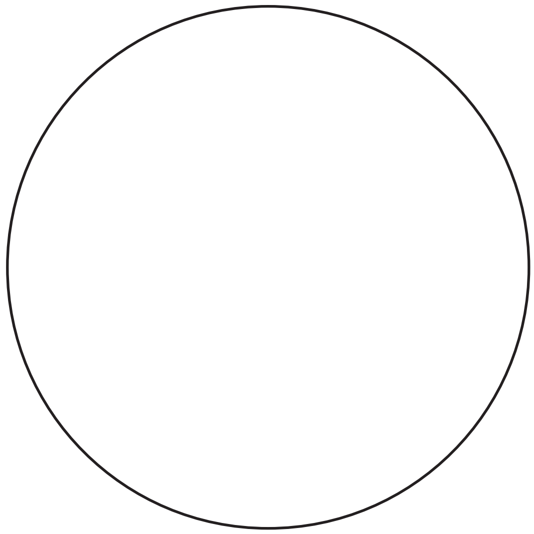
1.

2.

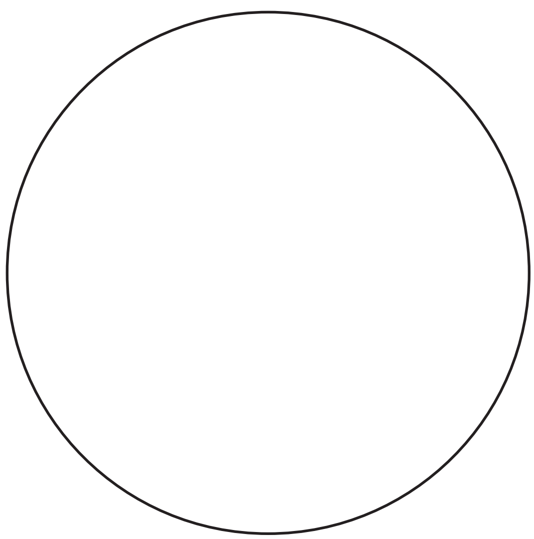
3.

Fraction Pictures

1. Shade 1/2 of the circle

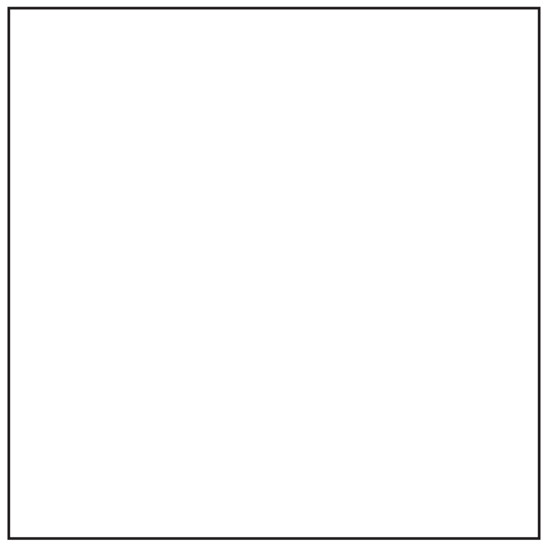


2. Shade 2/4 of the circle

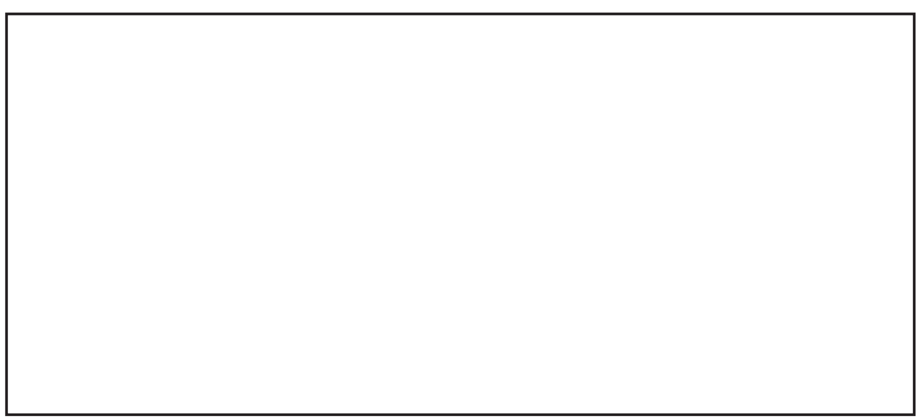


Fraction Pictures

3. Shade 1/4 of the square

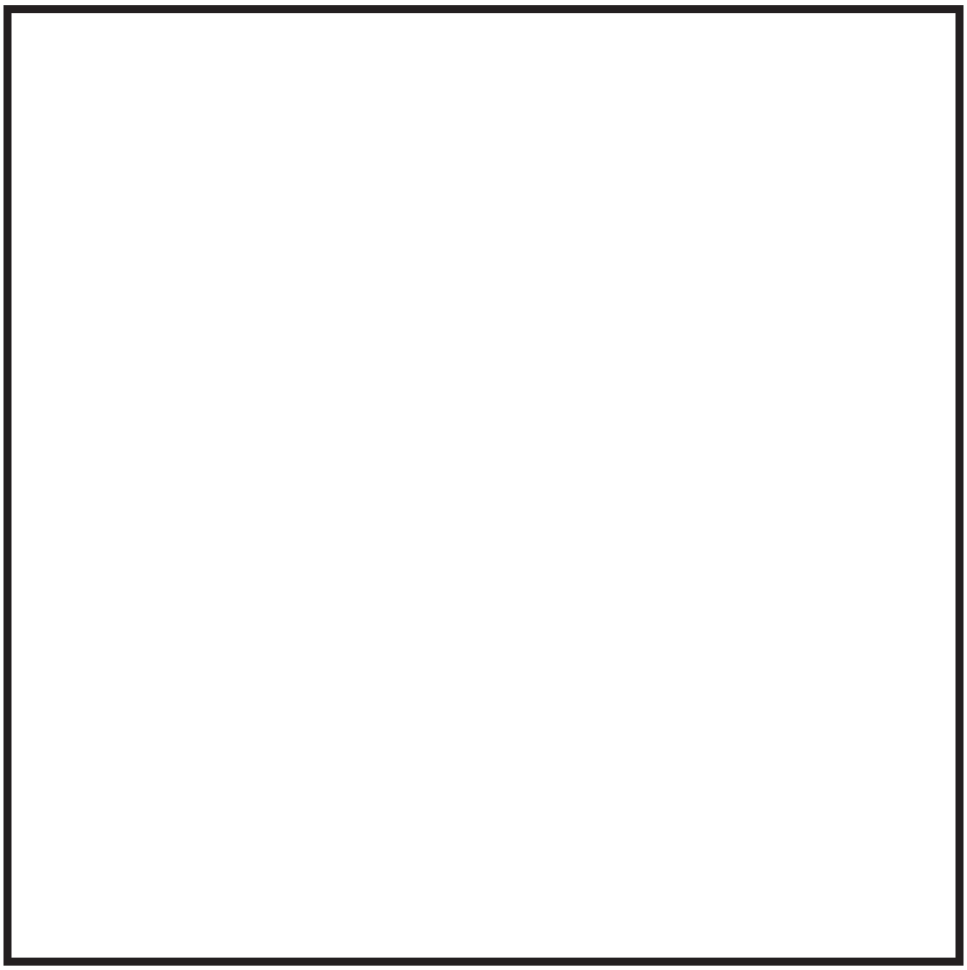


4. Shade 2/3 of the rectangle



Decimals

1. a. Divide the square (flat) provided into 10ths.

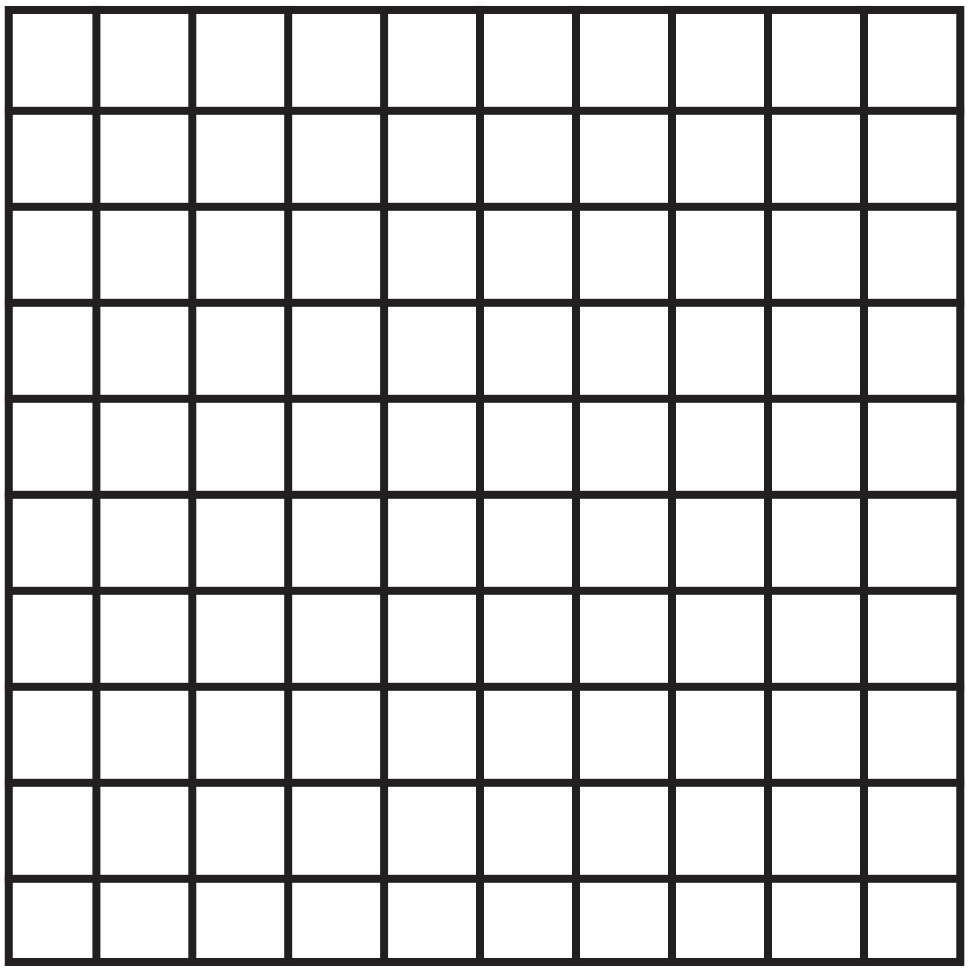


   b. Shade 3 tenths of the square.

   c. Write 3 tenths as a decimal.

Decimals

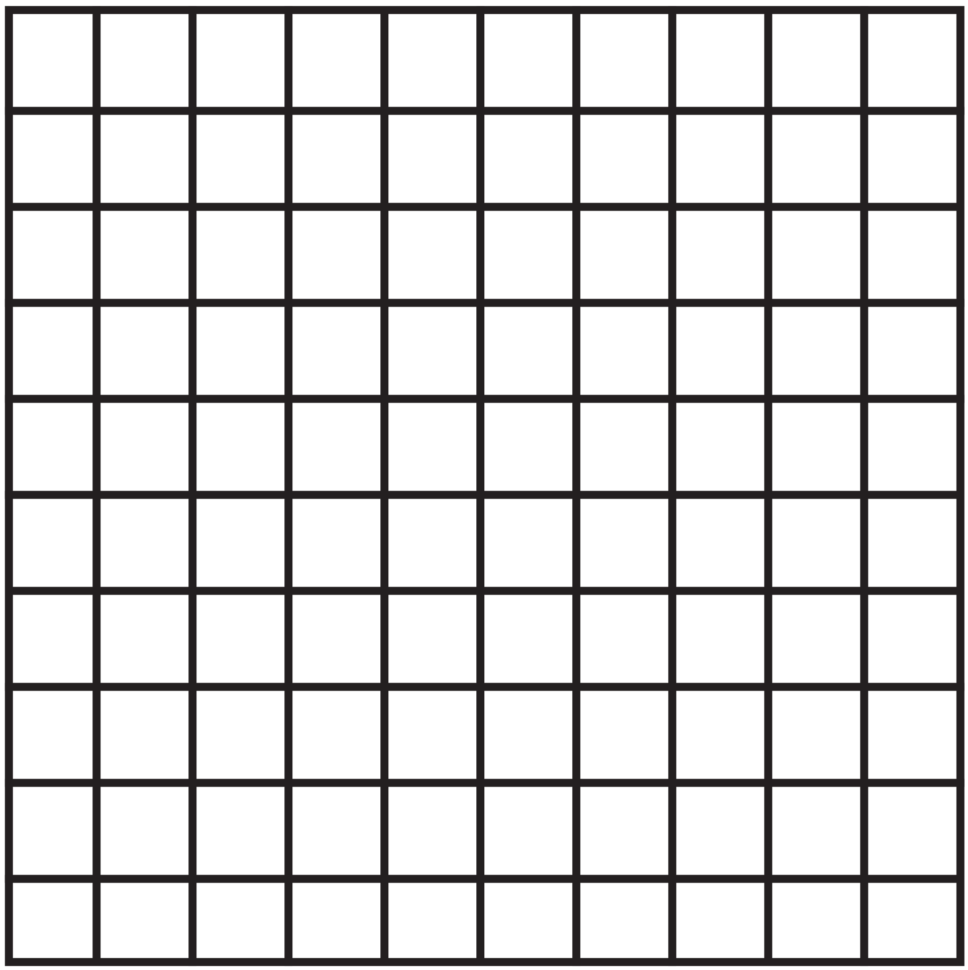
2. a. Using the flat provided, shade in 25/100.



b. Write 25/100 as a decimal.

Decimals

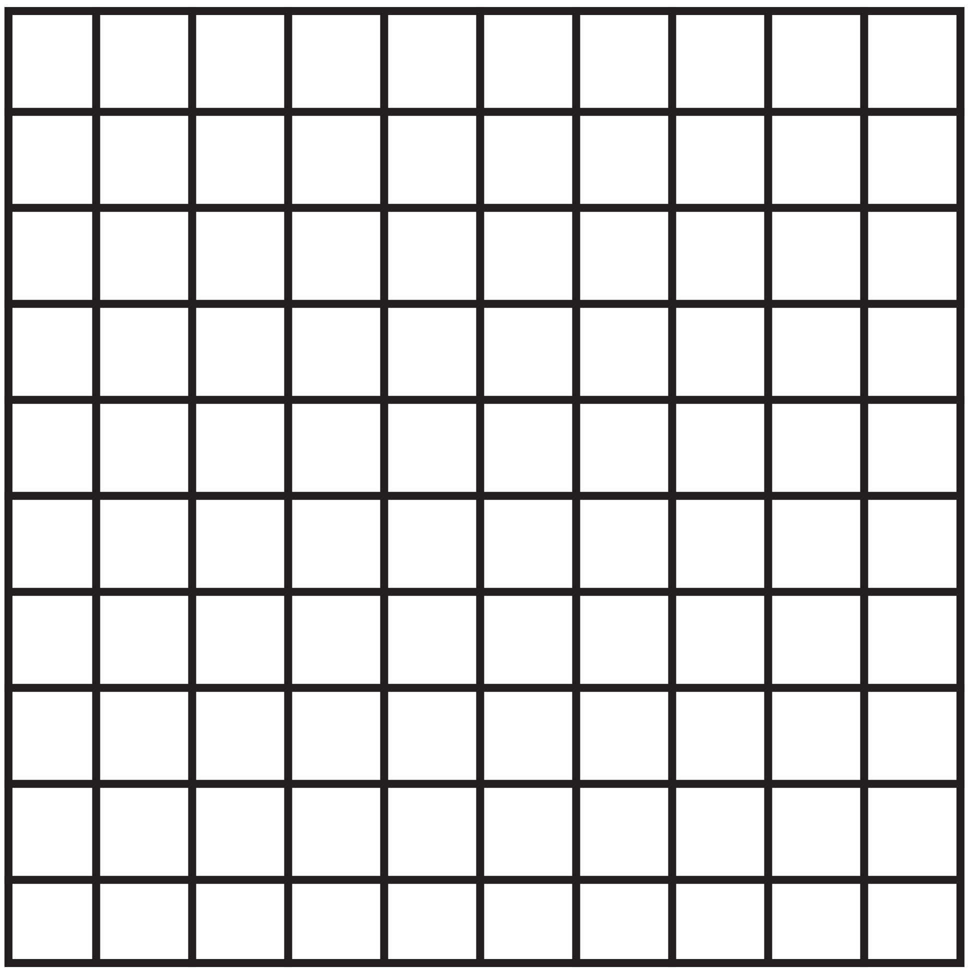
3. a. **Shade 5 hundredths.**



b. **Write as a decimal.**

Decimals

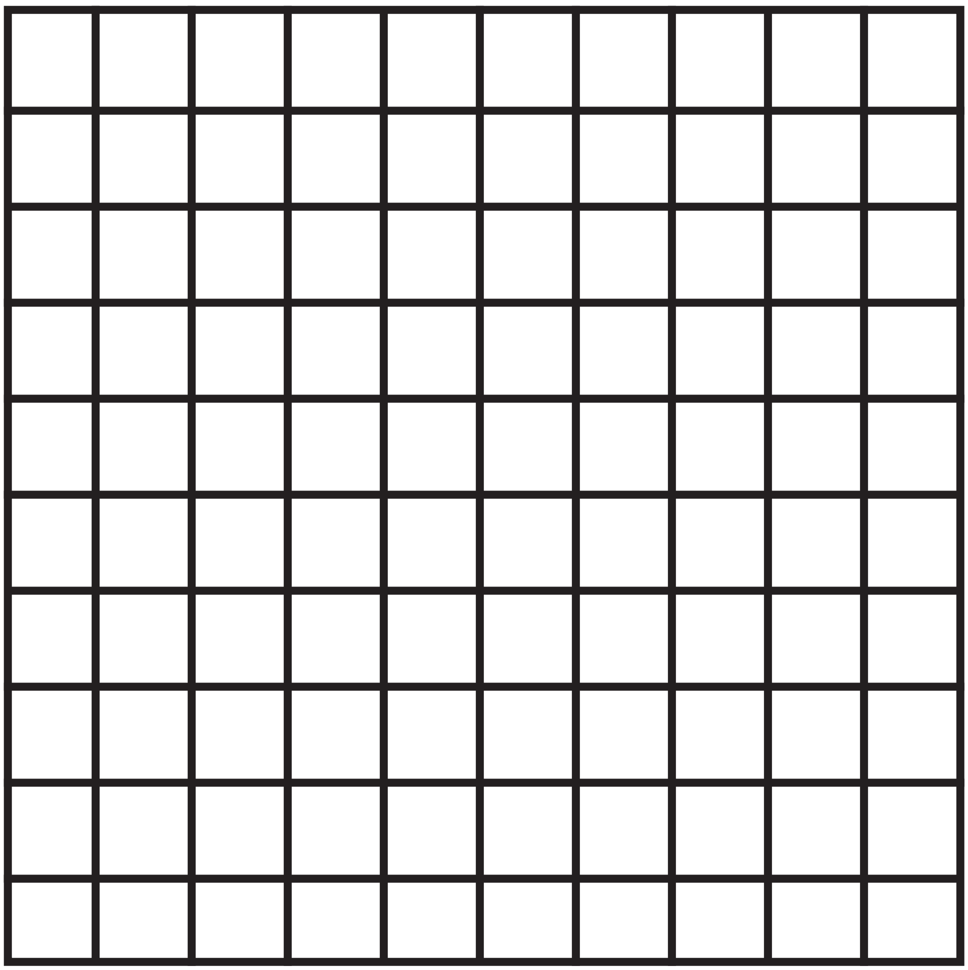
4. a. Shade 10 tenths.



b. Write as a decimal.

Decimals

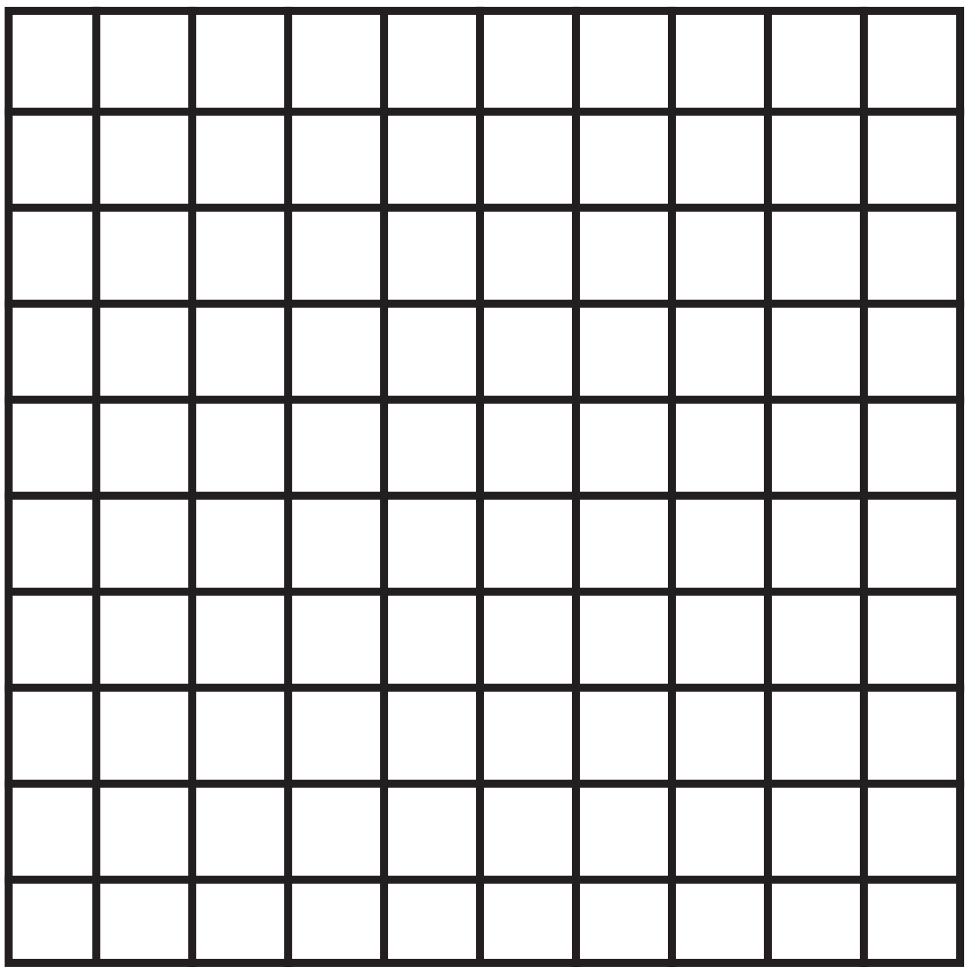
5. a. **Shade 10 hundredths.**



b. **Write as a decimal.**

Decimals

6. a. Shade 100 hundredths.

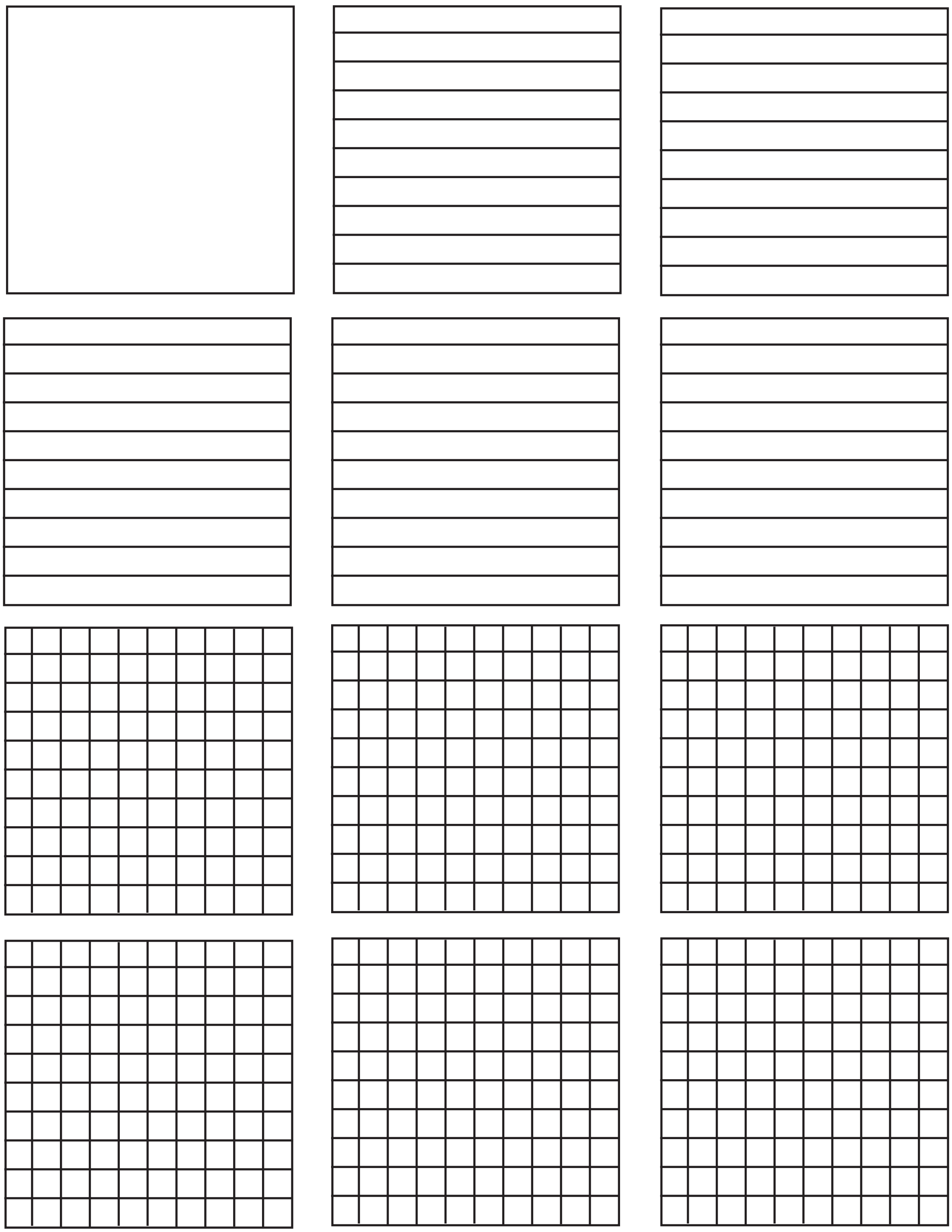


b. Write as a decimal.

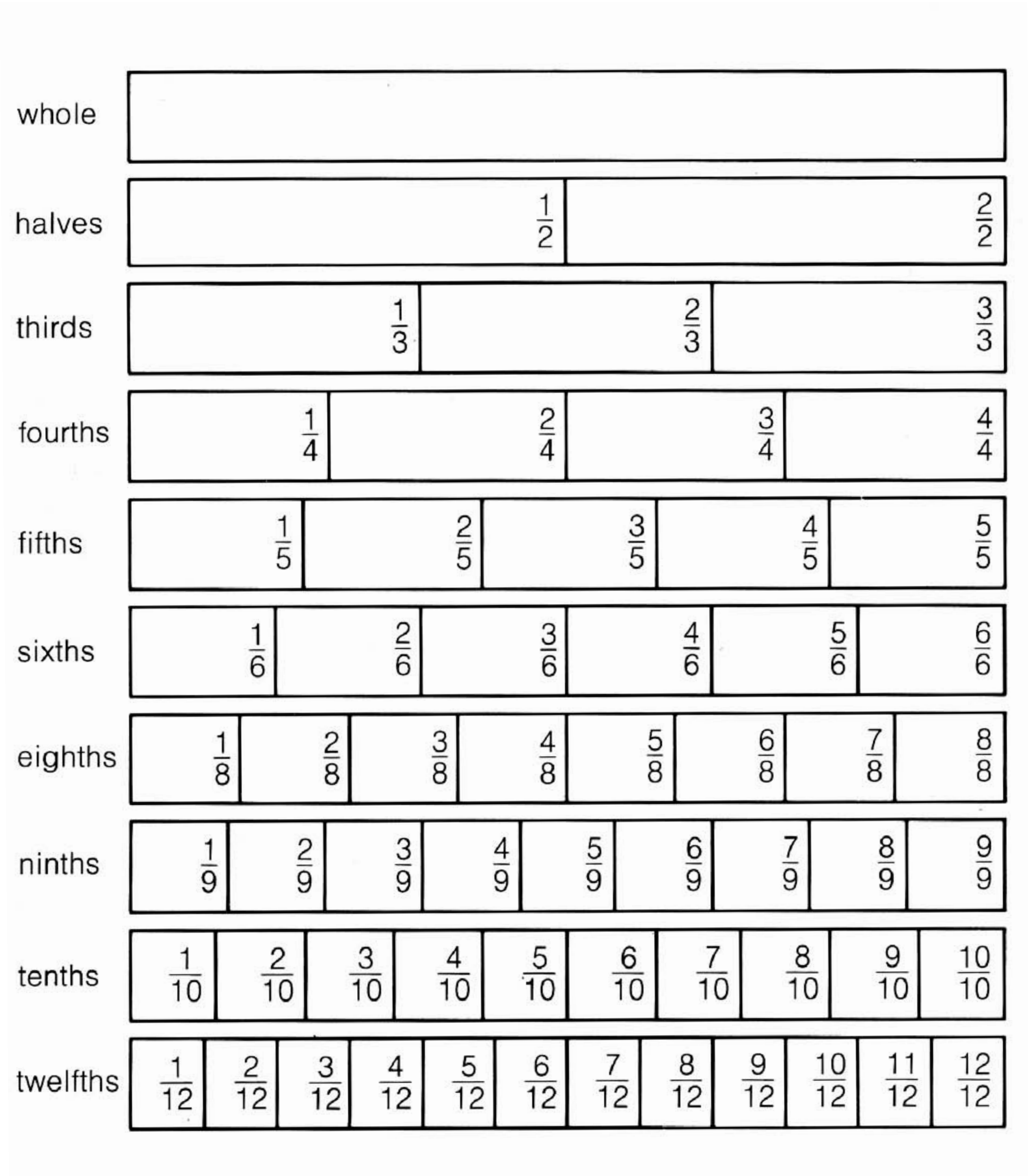
Makin' Change

Quarters	Dimes	Nickels	Pennies

Decimal Paper



Fraction Bars

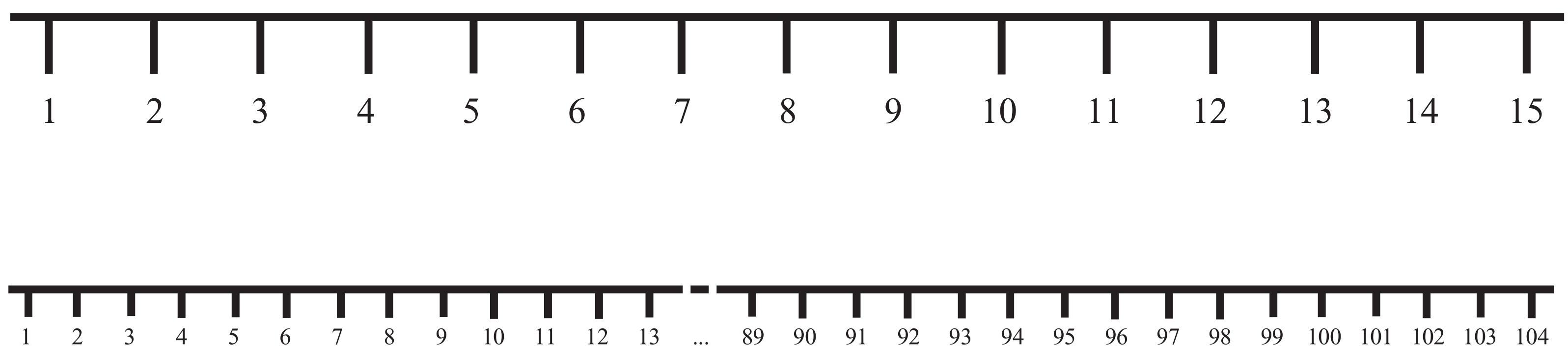


Comparison Conundrum

Compare the following using > or <.

1. 8 ___ 9
2. 11 ___ 21
3. 101 ____ 100
4. 1/3 ____ 2/3
5. 1/4 ____ 1/5
6. 2/3 ____ 2/8
7. 1/2 ____ 6/12
8. 0.1 ____ 0.2
9. 0.26 ____ 0.25
10. 0.1 ____ 1.0
11. 0.1 ____ 0.10

Number Line


1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
1 2 3 4 5 6 7 8 9 10 11 12 13 ... 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104

Module D
Transparencies

Module Goals
Module D: Equivalency and Number Comparison

The paraeducator will:

1. Use a variety of concrete materials to develop meanings for commonly used fractions and decimals (e.g., 1/4, 0.5, 1/3) for sets and wholes

2. Demonstrate equivalent forms of the same number through use of models, drawings, or other strategies

3. Compare numbers (equal, greater than, less than) using a variety of strategies

A Fraction

A *fraction* is the representation of part to whole.

The whole can be:

- one single object divided into parts, or
- a part of a set of objects.

Partitioning Sets and a Whole

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-

Examples of part of a set of objects:

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-
-
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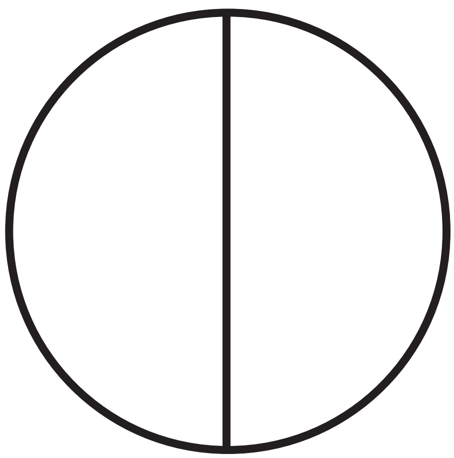
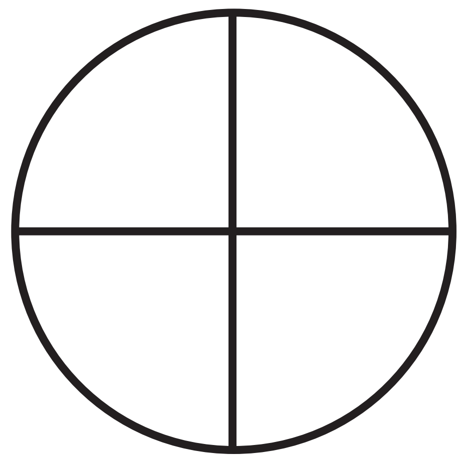
-
-
-
-
-
-
-
-

Fraction Pictures

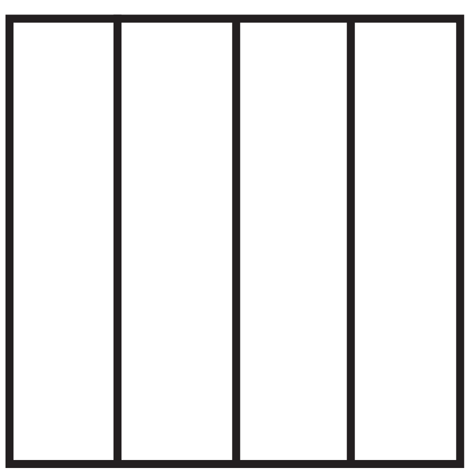
1.



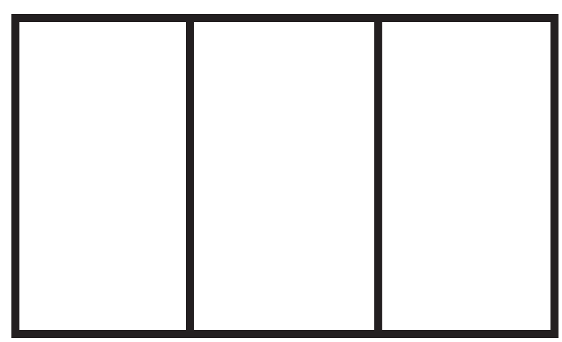
2.



3.



4.



Decimal Place Value Chart

Place value concepts for whole numbers.
Example:

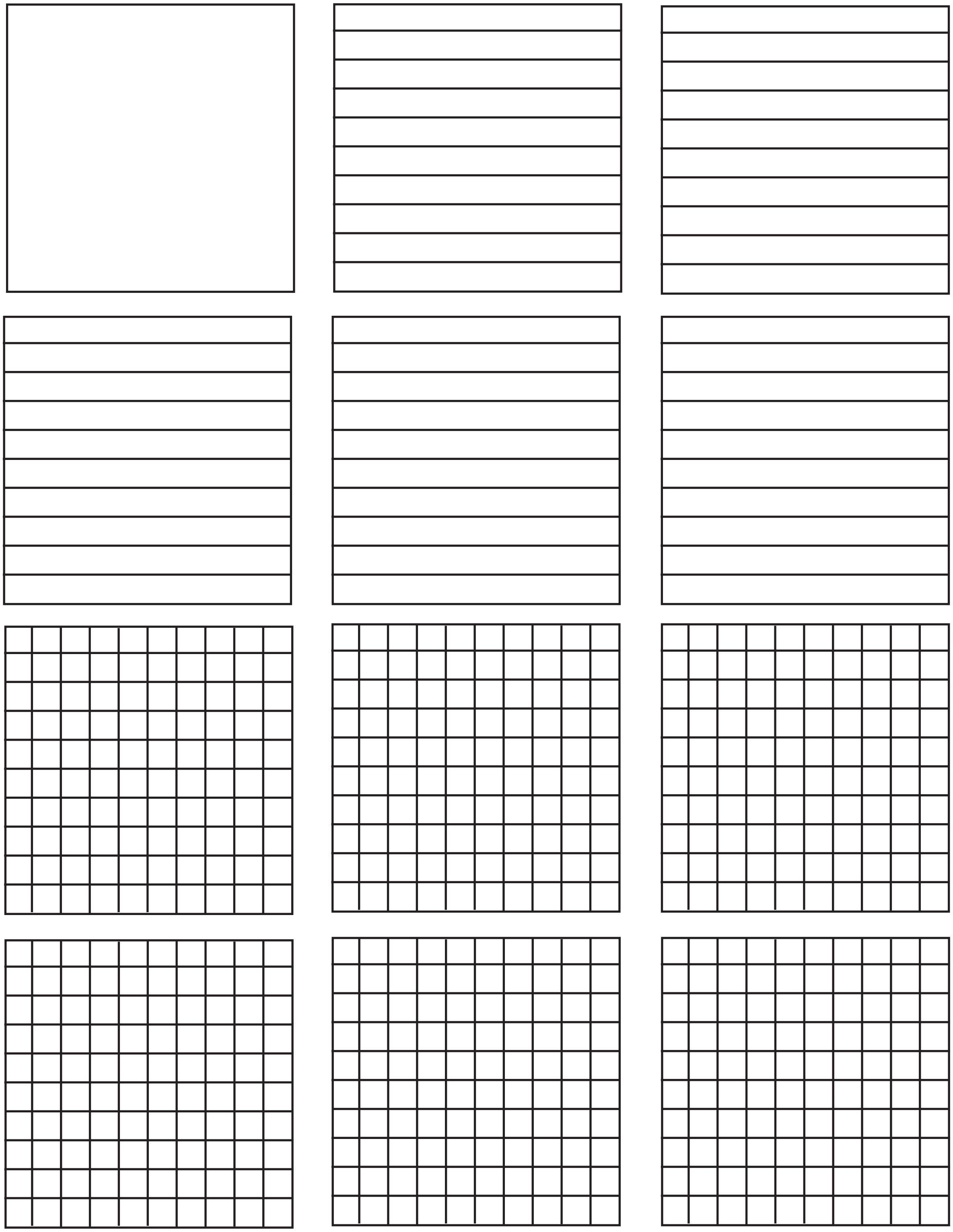
1 hundred + 2 tens + 5 ones = 125

Hundreds	Tens	Ones
1	2	5

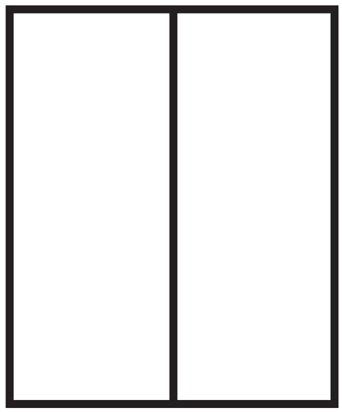
A decimal point is used to mark the division between wholes and parts.

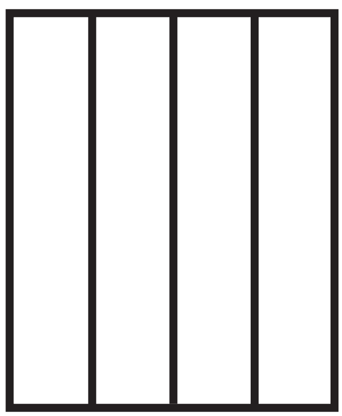
Hundreds	Tens	Ones	.	Tenths	Hundredths
1	2	5		3	7

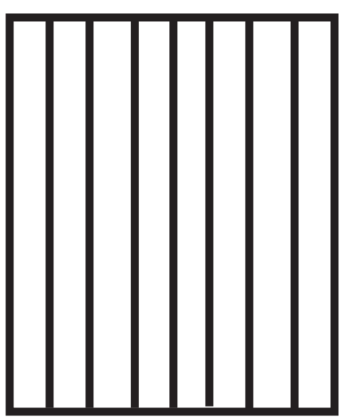
Decimal Paper



Equivalent Fractions





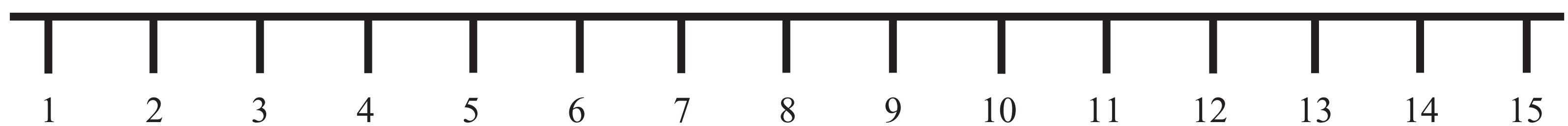


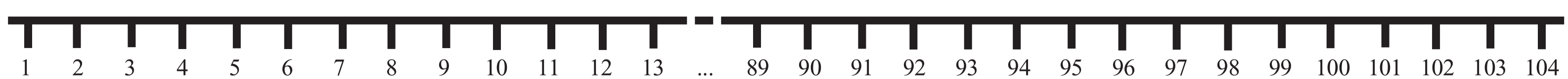
Comparison Conundrum

Compare the following using > or <.

1. 8 ___ 9
2. 11 ___ 21
3. 101 _____ 100
4. 1/3 _____ 2/3
5. 1/4 _____ 1/5
6. 2/3 _____ 2/8
7. 1/2 _____ 6/12
8. 0.1 _____ 0.2
9. 0.26 _____ 0.25
10. 0.1 _____ 1.0
11. 0.1 _____ 0.10

Number Line


1 2 3 4 5 6 7 8 9 10 11 12 13 14 15


1 2 3 4 5 6 7 8 9 10 11 12 13 ... 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104

Module E
Instructor's Guide

Module E:
Spatial Development and Measurement

A. Lecture: Module Goals

Use transparency and handout **Module Overview (T1/H1)** to review the goals of the module.

Module E: Spatial Development and Measurement
The paraeducator will:

1. Recognize and explore 2-D geometric shapes by their attributes (specific quadrilaterals, triangle, and circle; symmetry, diagonals, etc.)
2. Identify angle types (obtuse, right, acute)
3. Recognize and explore 3-D geometric shapes by their attributes (cube, sphere, cylinder, cone, prism and pyramid)
4. Solve problems using geometric relationships and spatial reasoning (e.g., congruence, similarity)
5. Use both non-standard and standard measurement for perimeter and area

Goal 1: Recognize and explore 2-D geometric shapes by their attributes (specific quadrilaterals, triangle, and circle; symmetry, diagonals, etc.).

1.1 Activity: Build It

Paraeducators will communicate math ideas by working with 2-D geometric shapes.

Materials:
- Pattern blocks or paper patterns for geometric shapes

1.1.1 Steps

- Divide class members into pairs and direct them to sit back to back.
- Each member of the class should have his or her own pattern blocks to work with.
- Direct one partner to be the "designer." Tell the designer to build a pattern or picture out of the pattern blocks. The other partner should not look at what the designer is doing.
- After building the pattern or picture, the designer describes it to his or her partner, who acst as the "builder." The builder attempts to replicate the pattern given the verbal directions. The following rules apply to the kind of information the designer can give the builder:
  - ▲ No colors may be used
  - ▲ Builder may not ask the designer questions
  - ▲ No vocabulary may be provided by the instructor or peers

- When the builder feels the work is completed, both partners should check their work, comparing it to the designer's pattern.
- Repeat the activity, switching roles, if time allows.
- Ask the class what would have made the activity easier and/or more accurate, looking for the following responses:
  - ▲ Vocabulary or common descriptors
  - ▲ Color name allowed for shape names not known
  - ▲ Possibly a grid with coordinates

1.2 Lecture: Describing 2-D shapes

Use transparency and handout **2-D Geometric Shapes (T2/H2).** Throughout this lecture participants should complete the chart to help them keep track of the 2-D shapes they will be reviewing and examining. The chart includes space to record the following: name of shape, a drawing (the color of the shape may also be recorded in this column; when using pattern blocks, the individual shapes have individual colors), number of sides, number of vertices (corners), number of angles, number of diagonals, lines of symmetry, and other (descriptors). Use the transparency as the first shape is presented (the square) to demonstrate how to complete the chart.

As the handout **2D Concepts (H3)** is discussed throughout this module, participants should draw pictures and make notes so that they will remember the concepts.

Begin with shapes in the patterns blocks. Start with the square as it is the easiest.

Use the **Intersections and Vertices** transcript **(T3)** and **2D Concepts** handout **(H3)** to review the following two new concepts.

Intersections and Vertices

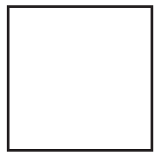
- *Intersection* – place where two or more lines cross
- *Vertex* (vertices is plural) – place where two lines (edges) intersect on a closed shape; corner

As a class, fill in the chart as listed for **name, drawing, number of sides, and number of vertices**.

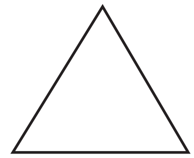
Review the following shapes. Ask participants to volunteer the information necessart to fill in the chart of 2-D Geometric Shapes. Use the handout **Shapes (H4)** if needed to aid in identifying shapes.

Shapes

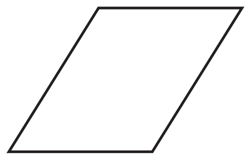
Square:



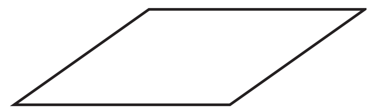
Triangle:



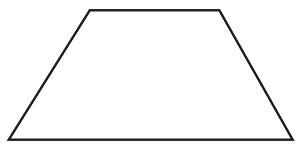
Rhombus:



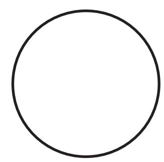
Parallelogram:



Trapezoid:



Circle:



Rectangle:

Name	Picture/Drawing	# Sides	# Vertices
Square	(Orange)	(4)	(4)
Triangle	(Green)	(3)	(3)
Rhombus	(Blue)	(4)	(4)
Parallelogram	(Tan)	(4)	(4)
Trapezoid	(Red)	(4)	(4)

Notice that if the description of the shapes were required by only using the number of sides and vertices, it would be confusing and difficult because four of the shapes have the same description. More information is needed.

The concepts of symmetry and diagonals are another simple way by which to classify these shapes.

Use the transparency **Symmetry and Diagonals (T4)** and the handout **2-D Concepts (H3)** to review the following two new concepts.

Symmetry and Diagonals

- *Symmetry* – a line of symmetry exists if the shape can be folded exactly on itself forming 2 equal parts
- *Diagonal* – a line that goes from vertex to vertex; can not cut a side

1.3 Activity: That's the Shape of Things

Paraeducators will define 2-D shapes by lines of symmetry and number of diagonals.

Materials:
- Paper for drawing and cutting
- Scissors
- Rulers
- **Shapes** handout **(H4)**

1.3.1 Steps

- Use the **Shapes** handout for this activity.
- Invite participants to use the paper and scissors provided to experiment with the listed shapes.
- Direct them to experiment with the definitions provided and add the data to the table.

Name	Picture/Drawing	Symmetry	# of Diagonals
Square	Orange	(4)	(2)
Triangle	Green	(3)	(0)
Rhombus	Blue	(2)	(2)
Parallelogram	Tan	(0)	(2)
Trapezoid	Red	(1)	(2)

Shapes vary greatly in this category, which makes it easier to identify them. Add the circle as an additional geometric shape. Fill in all categories for the circle.

Name	Picture/Drawing	Symmetry	# of Diagonals
Square	Orange	(4)	(2)
Triangle	Green	(3)	(0)
Rhombus	Blue	(2)	(2)
Parallelogram	Tan	(0)	(2)
Trapezoid	Red	(1)	(2)
Circle		Infinite (too many to be counted)	None

Goal 2: Identify angle types (obtuse, right, acute).

2.1 Activity: What's Your Angle

Paraeducators will use concrete objects to define three types of angles.

Materials:

- Pipe cleaners (1 per person)
- Transparency and handout **Angle Identifier (T5/H5)**
- Transparency **Angle (T6)**
- Handout **2-D Concepts (H3)**

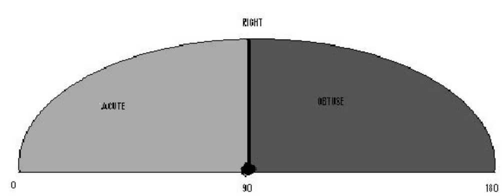
2.1.1 Steps

- Use the transparency and handout **Angle Identifier.**
- Introduce the term *angle.* Use the transparency **Angle** and handout **2-D Concepts.**

Angle

- For two intersecting lines, the measure of rotation about the vertex to bring one line on top of the other.
- Demonstrate the definition using a pipe cleaner.
- Assign participants to go around the room using their pipe cleaners to measure angles by bending the pipe cleaner to fit each side of the angle.
- When the angle has been shaped, place it on the **Angle Identifier** with one side on the bottom of the identifier pointing at zero and the vertex at the center.
- Record the object and its identification (acute, obtuse, right).
- Note that zero can be on either side; zero has been chosen to be on the left in this case only for convenience.

Angle Identifier



Use the transparency **Angle Concepts (T7)** and the handout **2-D Concepts**.

Angle Concepts

- *Degrees* – angles are measured in degrees
- *Acute* – an angle with a measure between and (not including)
- *Right* – "L" shape; exactly 90º
- *Obtuse* – an angle with a measure between 90º and 180º

2.2 Discussion: Further Classification

This discussion returns to the **2-D Geometric Shapes** handout and transparency **(H2/T2)** from goal 1.

For each of the shapes used or the pattern blocks, decide on the types of angle to help in identification. Add this information to the angle column of the chart.

Name	Picture/Drawing	# of Angles
Square	(Orange)	4 (all right angles)
Triangle	(Green)	3 (all acute)
Rhombus	(Blue)	4 (2 acute, 2 obtuse)
Parallelogram	(Tan)	4 (2 acute, 2 obtuse)
Trapezoid	(Red)	4 (2 acute, 2 obtuse)
Circle		No measurable angles
Rectangle		4 (all right angles)

Add the rectangle (not a pattern block) to the list and fill in the columns.
A rectangle has:

- 4 sides
- 4 vertices
- 2 lines of symmetry
- 2 diagonals

Note that the number of angles corresponds to the number of vertices.

Look at the data presented; note that some shapes are very similar to each other. By looking at the pattern blocks, ask which two are visually similar (parallelogram and rhombus).

In mathematics, we like to group objects together. For the shapes so far on the chart, look for other patterns within the data.

Starting with sides, introduce the term *quadrilateral* (shapes with 4 sides). Use the transparency **Quadrilaterals (T8)** and handout **2-D Concepts (H3).**

Quadrilaterals

Quadrilaterals are shapes with 4 sides.

Continue to use the handout **Shapes (H4)** introduced earlier as a reference for the following discussion and for identification purposes.

List the shapes that fit in the category of quadrilateral (square, rectangle, parallelogram, rhombus, trapezoid).

Which shape(s) do not fit anywhere? (circle, triangle)

To further classify, ask which one of the quadrilaterals is unlike any other (trapezoid). The trapezoid does not share one important piece: 2 sets of parallel lines.

Use the transparency **Parallel (T9**) and handout **2-D Concepts (H3)** to define the term.

- *Parallel – lines that remain the same distance apart; like railroad tracks or lines on notebook paper; never intersect (cross)*

A trapezoid only has one set of parallel lines. Add this to the *other* notes on the chart regarding trapezoids.

All other quadrilaterals may be classified under one of the remaining shapes. Discuss with the group the model for all others: the *parallelogram* is the general term for all remaining quadrilaterals. Use the transparency **Parallelogram (T10)** and handout **2-D Concepts (H3).**

- *Parallelogram* – quadrilateral with two sets of parallel lines; opposite sides are equal in length

All other shapes are special forms of the parallelogram. Discuss other descriptions that might help organize the shapes.

A rhombus looks very similar to a square. Notice it has more lines of symmetry than the parallelogram. Use the **More Parallelograms** transparency **(T11)** and the handout **2-D Concepts (H3).**

- *Rhombus* (also called a diamond) – a parallelogram with all sides equal

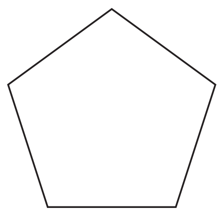
The square and the rectangle are virtually the same.

- *Rectangle* – parallelogram with four right angles
- *Square* – parallelogram with all sides equal and four right angles or a rectangle with equal sides

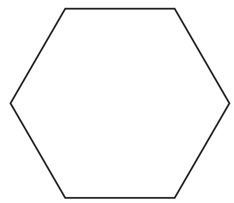
Note: Some texts vary the definition of trapezoid to say *at least* one pair of parallel sides which then makes all parallelograms a trapezoid. The definition used here states "only one pair" of parallel sides.

Now add *pentagons*, *hexagons,* and *octagons* to the discussion.

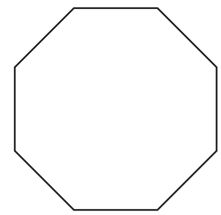
Pentagon:



Hexagon:



Octagon:



- ▲ Only use regular polygons where all sides and angles are the same

- Discuss how prefixes identify the number of sides
  - ▲ penta = 5
  - ▲ hexa = 6
  - ▲ octa = 8
- Also relate this to prior shapes; prefixes:
  - ▲ tri = 3
  - ▲ quad = 4
- Add information to the chart for the pentagon, hexagon, and octagon.
- A final thought is very important for classification. Use the transparency **Polygon (T12)** and the handout **2-D Concepts (H3):**
  - ▲ *Polygon* – can be defined by word parts
    - o *Poly* – many
    - o *Gon* – sided
    - o *Polygon* – many-sided *closed figure* (one continuous line that does not cross itself)
- Ask students which shape is not considered a polygon (circle). **All others are considered polygons.**

Goal 3: Recognize and explore 3-D geometric shapes by their attributes (cube, sphere, cylinder, cone, and pyramid).

3.1 Lecture: The Third Dimension

Many students struggle with the concept of three dimensional (3-D) shapes. It is important that students have hands-on experience with 3-D shapes as many students cannot visualize 3-D from a handout.

"Three dimensional" implies that the object has a physical height. Two-dimensional shapes, such as those used before, can be accurately represented on a sheet of paper because we only use two dimensions: length and width. Adding height to the description means that the shape would need to rise out of the paper. Being 3-D also implies that the shape could hold something such as liquid, or the shape has volume (discussed in later grades).

It is important that young students identify 3-D shapes in their environment such as cereal boxes, orange juice cans, or blocks of cheese. Young students must be able to identify basic 3-D shapes by sight and be able to describe their attributes.

For this goal 3-D shapes will be divided into two categories:

- polyhedra
- non-polyhedra

These terms may not be used until later grades but will be helpful for discussion. ***Polyhedra*** (plural for polyhedron) applies to **shapes that contain polygons** as listed for 2-D. (Review the definition of polygon from Goal 2.) For this exercise **any 3-D shape that contains circles** are considered *non-polyhedra*.

Remind class members to write these definitions in their math journals. Point out that these definitions apply to this activity and may defined in a broader way in other contexts.

Only polyhedra can be classified according to vertices, edges, and faces. Circles are complex concepts that are not fully defined until upper geometry and calculus. For non-polyhedra, any circular 3-D shape will be identified if possible by appearance and number of circles.

3.2 Activity: Seeing 3-D

Paraeducators will classify 3-D shapes by their attributes (faces, edges, vertices, circles).

Materials:

- Multiple sets of 3-D geometric shapes, OR
- Handout **3-D Object Patterns (H6)** per group (if no geometry sets available)
- Scissors and glue (or tape) if building shapes

3.2.1 Steps

- Divide the class into small groups of 3-5 each.
- Hand out sets of 3-D geometric shapes to each group.
- As a class, define the following terms. Use the transparency and handout **3-D Concepts (T13/H7).**
    - ▲ *Faces* – flat polygonal (shaped like polygon) surface
    - ▲ *Edges* – place (line) where two faces come together
    - ▲ *Vertices* – (same as 2-D); points or corners where two edges meet
- Direct the groups to divide the shapes into polyhedra and non-polyhedra categories. Ask them to refer to their written definitions if needed.
- Use handout **3-D Shapes (H8).**

- Using a set of geometric shapes, ask each group to attempt to match the shape (considering faces, edges, vertices) with the chosen list of names listed on the handout:
  - ▲ Sphere
  - ▲ Rectangular prism
  - ▲ Cube
  - ▲ Cone
  - ▲ Pyramid
  - ▲ Cylinder
- Remind students to draw the shape in the space provided for later reference. Students will need assistance with drawing 3-D objects. The instructor may want to provide samples of shapes without their labels as models for drawing.

Polyhedra

Name	Picture/Drawing	# of Faces	# of Edges	# of Vertices	Real-Life Examples
Rectangular Pyramid	5	8	5	*Egypt*	
Cube		6 (all same)	12	8	*Dice*
Rectangular	6	12	8		*Butter stick*
Prism					

Non-polyhedra

Name	Picture/Drawing	# of Circles	Real-Life Examples
Cylinder		2	*Frozen orange juice; roll of coins*
Cone		1	*Ice-cream cone*
Sphere		NA	*Earth; ball*

- Check the answers of the group and give hints, if necessary, for drawing the shapes.
- Share as many real-life examples as possible.
- Relate 2-D shapes to their 3-D partners:
  - ▲ Circle and sphere (stacked, graduated circles) or cylinder (like a stack of circles)
  - ▲ Square and cube
  - ▲ Rectangle and rectangular prism
  - ▲ Cylinder (circle and a rectangle cut apart)
    - o Can demonstrate by simply rolling up a rectangular sheet of paper to make a cylinder
  - ▲ Pyramid and triangle (have a square or rectangle for bottom)
  - ▲ Cone (this is difficult – may include circle, possibly triangle)

Goal 4: Solve problems using geometric relationships and spatial reasoning (e.g., coordinate geometry, congruence, similarity).

4.1 Activity: Tricky Tangrams

Paraeducators will use tangram patterns to solve spatial reasoning problems.

Material:

- One set of tangram pieces for each class member
- Use the handout **Tangram (H9)** as a model to create tangrams for each member. Provide this handout for each participant for personal use at a later date. It is not necessary at this time except as a model for the instructor to make tangrams for each participant ahead of time
- Use the handout and transparency **Tricky Tangrams (H10/T14)**

4.1.1 Steps

- Begin the activity by explaining that many students, while weak in arithmetic steps, are very visual and would prefer to solve problems using their visual strengths.
- Point out that spatial reasoning is an important step in solving many problems, beginning with interpreting the goal of the problem by drawing a diagram.
- Provide participants with the handout listed above and use the transparency **Tricky Tangrams.**
- Direct individual participants to use their tangram pieces to build the listed shapes on the first page of the **Tricky Tangrams** handout.
- Make sure that participants also use their math journals to sketch and record the following solutions:

  Tricky Tangrams

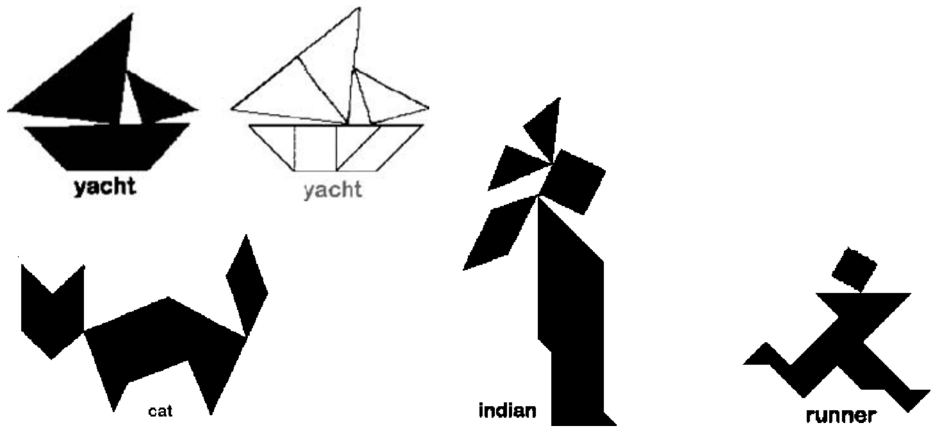
  Use your tangrams to create the following shapes:

  ▲ Use 2 pieces to create a triangle
  ▲ Use 2 pieces to create a square
  ▲ Use at least 3 pieces to create a parallelogram
  ▲ Use 3 pieces to make a trapezoid
  ▲ Use 3 pieces to make a trapezoid

- Ask class members to create clue challenges (similar to those above) as time allows to share with a partner.
- Hand out tangram pattern outlines and allow participants to build in the patterns on the handout **Tricky Tangrams.** This is purely spatial practice with no lines given.

Tricky Tangrams: pages 2 and 3

Use your tangrams to create the following shapes. The first gives you a model. The rest are for you to figure out!



4.2 Activity: Same or Similar?

Paraeducators will define congruence and similarity through models.

Materials:

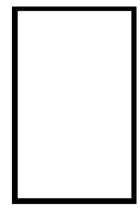
- Use the handout and transparency **Graph Paper (H11/T15).**
- Pattern blocks
- Crayons (markers, etc)
- Straight edge
- Pipe cleaners

4.2.1 Steps

- Trace a 2x3 rectangle on the overhead graph paper.
- Ask participants to replicate the rectangle on their graph paper.
- Ask them to draw one that is *exactly* the same.
- Ask what makes the rectangles the same.
  - ▲ Width (height)
  - ▲ Length
  - ▲ Area (amount of squares inside)
- Explain that objects that are exactly the same are ***congruent.*** Use transparency and handout **Congruent and Similar (T16/H12).** ***Congruent objects*** have:
  - ▲ the same angle type and measurement
  - ▲ identical length of sides
- Ask class members to draw another congruent rectangle with a different orientation, This may be a little confusing at first; you may need to explain that

the shape or objects are still congruent even if they have been rotated or moved. Example: Samples A and B are congruent rectangles.

A. 

B. 

- Direct students to pick a pattern block and trace it on their graph paper.
- Have them trade their drawing with a partner and ask the partner to create a free-hand drawing of a congruent shape on the graph paper.
- Trace a new 2x3 rectangle on the overhead.
- Ask the class to replicate the rectangle on the overhead and then draw a new rectangle that is exactly twice as big.
- Ask if they are congruent (NO, they are not because they are not exactly the same).
- Ask what is the same about the two rectangles:
  - ▲ Same shape
- Ask what changed:
  - ▲ Width (height)
  - ▲ Length
  - ▲ Area (amount of square inside)
- Although the above changed, the length and width should have increased by twice the amount (2x3 became 4x6).
- The difficulty comes in the *area* (2x3=6 but 4x6=24). The individual length and width increased by twice the amount, but the area increased by much more. It increased by 4 times.
  - ▲ *Area* – amount of space inside a closed figure (remind participants to record this definition)
  - ▲ The width and length changed by twice as much. However, since ***each*** dimension of the rectangle changed by twice as much, the result is 2x2 = or, 4 times as much
- These rectangle shapes are ***similar*** but not congruent. To be ***similar*** :
  - ▲ All angles must remain equal
  - ▲ The relationship of the sides must be *proportional* (the ratio of their dimensions must be the same)
    - o 4/2 = 2 and 6/3 = 2
- Direct participants to use pipe cleaners to confirm that the angles are equal between shapes.
- Try the same exercise for similarity with several of the pattern blocks. Angled shapes are more difficult.
- Start with straight edges.
- Also look at the height (altitude) of objects
- Have pairs of participants decide on the number of times to enlarge or shrink an object (if not using pattern blocks).

- Remind them that each dimension must increase by the same multiple (e.g., 3 times, 4 times).

Goal 5: Use both non-standard and standard measurement for perimeter and area.

5.1 Lecture: Non-Standard and Standard Measurement

Similar to algorithms, measurement can be forced too early toward standard measurement. Young students need to see the purpose of standard measurement before using rulers.

A starting point is using non-standard measurement. For non-standard measurement, we can use any object available to express the size of an object. This includes paper clips, hands, or pencil lengths. These measures are called non-standard because not everyone has hands the same size, uses the same size paper clips, or has pencils that are the same length. Students need to practice measuring objects in their environment with a variety of non-standard measures. Special attention should be paid to the difference in measurements throughout the experience. It is important to see that to communicate, we need an agreed-upon measurement system. Traveling is a good example. Measuring in paper clips from one city to another could be difficult and inaccurate as opposed to measuring in agreed-upon miles.

Once the need for uniform measurements is established, English and metric measurement units may be introduced. English Standard (U.S.) is what we commonly use in the United States (e.g., miles, inches, feet, gallons). The Metric System is used throughout the rest of the world and in the scientific community. Measurement links together many basic concepts: basic operations, fractions, decimals, and spatial reasoning. Students need repeated practice with both systems of measurement.

Make sure to discuss the appropriate abbreviations for the units.
Inches = in.

- We only use inches. There is not an abbreviation for half-inch, etc., because that is part of the measurement itself.

Feet = ft.
Centimeter = cm

- *Note that no periods are used for the abbreviation in the Metric System.*

5.2 Activity: Measure It

Paraeducators will use non-standard units to measure common classroom objects.

Materials:

- Objects for measuring, or class can use anything in the room
- Pattern blocks

5.2.1 Steps

- Have every class member choose a unit of measure (key, pencil, paper clip, shoe, etc.). Define for the class that whatever they pick is their particular unit of measurement and the measurement of objects will be recorded as such; for example, "a piece of paper measured 6-1/2 key lengths."
- Decide as a class or as a small group what four objects they will each measure with their individual unit of measurement.
- Direct them to proceed to measure objects with their individual units of measure (e.g., measure the white board in paper clips and shoe lengths and record that measure).
- Make sure to label each measurement (12-1/2 shoe lengths or 5 paper clips).
- Compare the measurements as a group/class.
- Ask who is more correct.
- Answer: No one is more correct because we have not agreed on the measurement tool.
- Explain that this is ***non-standard measurement.***
- *Measurement* implies describing size or amount. Remind participants to record this information in their math journals.
- Explain that for young children, simply organizing their planning and counting is a good basis for measurement.
- Ask what makes communication about measurement difficult.
  - ▲ Not standard – a standard unit is needed to accurately discuss the size of the object
- Discuss what would happen if we all decided on a shoe to be the unit of measure
  - ▲ Not all shoes are the same length (men vs. women)
- Note that we not only need to choose an object, but the size of the object as well – this is the need for ***standard measurement***

5.3 Activity: Make It Standard

Paraeducators will use a standard ruler to measure in both the English and Metric systems.

Materials:
- One ruler per person (with both U.S. and Metric systems)
- Meter stick (if available)
- Yard stick (if available)

5.3.1 Steps

- Explain that measurement involves a variety of skills (basic operations, fractions, decimals, and spatial reasoning).
- Point out that one of the most difficult skills is translating all of the marks on the ruler.

- Remind class members that students in grades K-4 must know U.S. units to the quarter inch and Metric to the centimeter.
- Starting with the U.S. system, make sure everyone is looking at the correct side of the ruler.
- Point out that the *inch* is the largest unit on the ruler – find the *inch.*
  - ▲ Numbered 1-12
  - ▲ Ask where the ruler starts – zero, which is actually the end of the ruler but not marked
  - ▲ Ask if they know what 12 inches are called – *one foot*
  - ▲ The standard ruler shows 1 foot
  - ▲ If the yard stick is available, see how many inches there are in one yard (36 inches or 3 feet)
- Have each person find something that is 1 inch long (body part, clothing, school supply, etc.) and record that object along with drawing a line that is one inch long and labeling it.
- This comparison to a standard unit is called a ***benchmark.***
  - ▲ Benchmarks are useful when the actual ruler is not present
- Discuss the *half inch.*
  - ▲ Think back to fraction strips – a half divides the area into two equal pieces
  - ▲ Half inch is the long line between two numbered inches
  - ▲ If we start at 1, and then count the number of little lines up to 2, we get 16 lines – the 8th line is the half inch line
- Repeat the benchmark exercise to a half inch – participants should start from zero to prevent confusion.
- Should be half as long as the inch line drawn
- For half inches, show how to write those that are between numbers.
  - ▲ The half inch between 1 and 2 is called *one and one-half inch* or 1-1/2 inch
  - ▲ The "and" mean plus so 1-1/2 implies 1 inch and a little more
  - ▲ Labels are required to note the standard unit of *inch*
- Find the quarter inch or 1/4 inch.
  - ▲ From our fraction knowledge, we should have four equal parts
  - ▲ We also know that 1/4 is half of one half
  - ▲ The quarter inches are halfway between a numbered inch and the half mark
  - ▲ They are shorter lines than the half but longer than the others
  - ▲ Starting at zero, we can count at every 4, there should be a longer line (count the half again as well)
- Find a benchmark and record with a length of line.
- Work on writing quarter inches as was practiced with the half.
  - ▲ 1/4, 2/4 = 1/2, 3/4, 4/4 = 1 *inch*
  - ▲ Practice writing different combinations such as between 3 and 4, 0 and 1, etc. (e.g. 3-1/4 in., 2-3/4 in., 5-1/2 in.)
- Use the other side of the ruler for Metric (often labeled MM).
- Metric can be confusing because the MM stands for millimeter, which is the

smallest line (not the numbered lines).

- For K-4 we only focus on *centimeter,* which are the numbered lines.
- **DO NOT let students believe that there are 30 centimeters** (end of the ruler) to one foot – higher grades use conversions for Metric to U.S., which require decimals.
- Note that the first mark of the metric units is not the end of the ruler – it is the first long mark (zero).
- The most important concept is the prefix *centi-*.
  - ▲ Discuss that this is similar to *cent* in our money system, which means 100 pennies in a dollar
  - ▲ There are 100 centimeters in a meter stick (if available)
- Centimeters are easy to measure as we just report the number listed.
- Find a benchmark for the centimeter and draw the line with a label.
- For practice, individually measure the sides of each pattern block (may wish to trace and then label on the diagram) for the following:
  - ▲ Nearest inch
  - ▲ Nearest half inch
  - ▲ Nearest quarter inch
  - ▲ Nearest centimeter
- Compare answers as a group to check accuracy.
- Debrief as necessary.
- If appropriate, discuss that the geometric properties of opposite sides are of equal length for parallelogram, rhombus and square (part of definition), and that the triangle has equal sides (*equilateral triangle).*
- If there is time, discuss *perimeter* (total measure of the outside edges by adding all amounts).

6.1 Assessment

Paraeducators will <u>use their notes and handouts</u> to assist them in an assessment of the Assisting with K-4 Mathematics in the Classroom Academy.

Use handout **Final Assessment, Assisting with K-4 Mathematics in the Classroom (FA).**

Allow **<u>60 minutes</u>** for the assessment. After attendees have completed the assessment, ask them to complete the course evaluation and other instructor-provided information. Use the answers provided in the grading rubric to assist in grading the assessment.

Note to Instructor: Prior to beginning the assessment, determine how to inform class members of test results. **Do not return** the hard copy of the test to class members. Return the hard copies of individual tests to the PARA2 Center with other Academy-related materials.

Module D:

1. **Using the following fractions, circle the number representing the "whole" and draw a box around the number representing the "part." (9 points, 3 points per problem)**

   Answer:

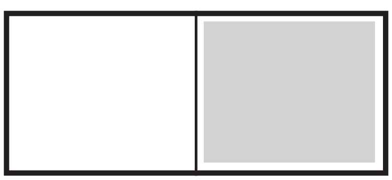
   a. $\boxed{1}/4$ **b.** $\boxed{5}/8$ **c.** $\boxed{3}/5$

2. **Divide the following rectangles and shade the parts as requested. (20 points, 5 points per problem)**

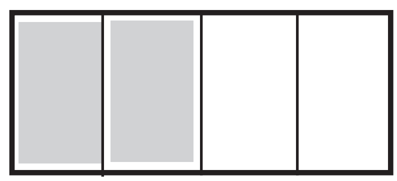
   The following answers *contains the most common responses to these problems*. If the answer given by the student is different, *the instructor must check* to see if it is accurate. There is more than one possible response per problem.

   Answer:

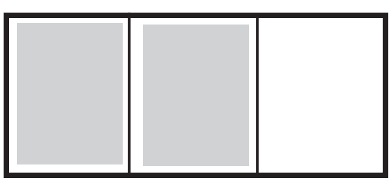
Divide and shade 1/2



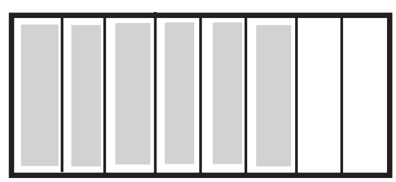
Divide and shade 2/4



Divide and shade 2/3



Divide and shade 6/8



Instructor Grading Key

Module A:

1. Describe how Reys, Suydam and Lindquist (1992) define mathematics. (5 total points, 1 per answer for the following 5 responses)

Answer:

Mathematics is …

1. **a study of patterns and relationships**
2. **a way of thinking**
3. **an art (involves creativity – not just rules)**
4. **a language**
5. **a tool (used in almost everything we do)**

Module C:

Use only addition and subtraction signs to fill in the empty boxes. The following answers *contain the most common responses to these problems*. If the answer given by the student is different *the instructor must check* to see if it is accurate. There is more than one possible response per problem. (20 points, 5 points per problem)

Use only addition and subtraction signs to fill in the empty boxes:

5	+	7	+	3	+	1	-	4	=	12
Another possible answer:	+		-		-		+			

20	-	9	+	1	-	7	=	5

Use only multiplication and division signs to fill in the empty boxes.

2	**x**	6	÷	4	**x**	1	=	3

30	÷	5	**x**	4	÷	3	÷	2	=	4

3. **Express the following as numbers: (4 points, 2 points per problem)**

   a. 4 hundreds + 6 tens + 9 ones = _________ **answer: 469**

   b. 8 hundreds + 0 tens + 3 ones = __________**answer: 803**

4. **Write the following as decimals: (6 points, 2 points per problem)**

   a. 3/10 = __________**answer: .3**

   b. 1/4 = __________ **answer: .25**

   c. 35/100 = __________ **answer: .35**

5. **Write the appropriate sign in the blanks below. Use >, <, or =. (8 points, 1 point per problem)**

   a. 192 ___<__ 203

   b. 14 ___>___ 3

   c. (1 x 4) ___=___ 4

   d. 5/8 ___>___ 3/8

   e. 2/3 ___>___ 2/8

   f. 0.2 ___<___ 0.3

   g. 0.27 ___>___ 0.26

   h. 0.1 ___=___ 0.10

Module E

Define the following: (8 points, 2 points per problem)

1. What is a vertex?
   Answer:
   A vertex is a place where two lines (edges) intersect on a closed shape; a corner

2. How many vertices are there in a square?
   Answer: 4

3. How many vertices are there in a triangle?
   Answer: 3

4. What kind of angle is exactly 90°?
   Answer: a right angle

5. Place a check mark in front of each of the following shapes that is a quadrilateral:
   (8 points, 1 point per problem)

   a. ___**x**___ square

   b. ______ triangle

   c. _____ **x** __ rhombus

   d. ______ hexagon

   e. _____ **x** __ trapezoid

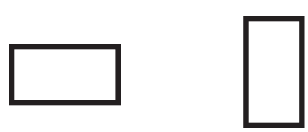
   f. ______ circle

   g. ___ **x** ___ rectangle

   h. _____ **x** _____ parallelogram

6. Are the following shapes congruent? Circle Yes or No. (4 points, 2 points per question)

a. Yes or No



b. Yes or No

7. Multiple Choice:

Write the letter that best fits the provided definition on the space provided. (8 points, 4 points per problem)

1. Measurement using objects found in the environment is called: ____**b.**_______

   a. standard measurement
   b. non-standard measurement
   c. metric measurement
   d. English measurement

2. Comparison of an object used for measurement to a standard unit of measurement is called: ______**b**._______

   a. measuring by feet
   b. a benchmark
   c. standard measurement
   c. measuring in centimeters

Module E
Handouts

Module Goals

Module E: Spatial Development and Measurement

The Paradeducator will:

1. Recognize and explore 2-D geometric shapes by their attributes (specific quadrilaterals, triangle, and circle; symmetry, diagonals, etc.)
2. Identify angle types (obtuse, right, acute)
3. Recognize and explore 3-D geometric shapes by their attributes (cube, sphere, cylinder, cone, prism and pyramid)
4. Solve problems using geometric relationships and spatial reasoning (e.g., congruence, similarity)
5. Use both non-standard and standard measurement for perimeter and area

2-D Geometric Shapes

Name of Shape	Picture/ Drawing of Shape	# of Sides	# of Vertices (Corners)	# and Type of Angles	# of Diagonals	Lines of Symmetry	Other

2-D Geometric Shapes

Name of Shape	Picture/ Drawing of Shape	# of Sides	# of Vertices (Corners)	# and Type of Angles	# of Diagonals	Lines of Symmetry	Other

2-D Geometric Shapes

Name of Shape	Picture/ Drawing of Shape	# of Sides	# of Vertices (Corners)	# and Type of Angles	# of Diagonals	Lines of Symmetry	Other

2-D Geometric Shapes

Name of Shape	Picture/ Drawing of Shape	# of Sides	# of Vertices (Corners)	# and Type of Angles	# of Diagonals	Lines of Symmetry	Other

2-D Concepts

Intersection
Place here two or more lines cross

Vertex
(vertices is plural)
Place where two lines (edges) intersect on a closed shape; corner

Symmetry
A line of symmetry exists if the shape can be folded exactly on itself forming two equal parts

Diagonal
A line that goes from vertex to vertex (it cannot cut a side)

Angle
For two intersecting lines, an angle is the measure of rotation about the vertex to bring one line on top of the other.

- ***Degrees*** – angles are measured in degrees
- ***Acute*** – an angle with a measure between 0° and 90° (not including 90°)
- ***Right*** – "L" shape; exactly 90°
- ***Obtuse*** – an angle with a measure between 90° and 180°

Quadrilaterals
2-D shapes with 4 sides

Parallel
Lines that remain the same distance apart like railroad tracks or lines on notebook paper (they never intersect; cross)

Parallelogram
Quadrilateral with two sets of parallel lines. The opposite sides are equal in length.

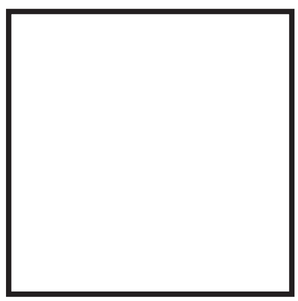
- ***Rhombus* -** (also called a diamond) a parallelogram with all sides equal
- ***Rectangle* -** a parallelogram with 4 right angles
- ***Square* -** a parallelogram with all sides equal and 4 right angles or a rectangle with equal sides

Polygon
- *Poly* - many
- *Gon* – sided
- Many-sided *closed* figure (one continuous line that does not cross itself)

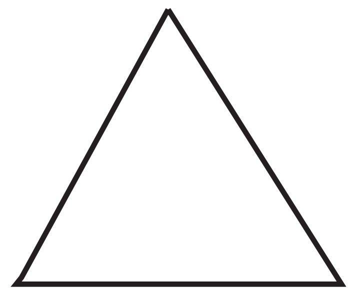
- ***Pentagon*** – 5-sided figure
- ***Hexagon*** – 6-sided figure
- ***Octagon*** – 8-sided figure

2-D Shapes

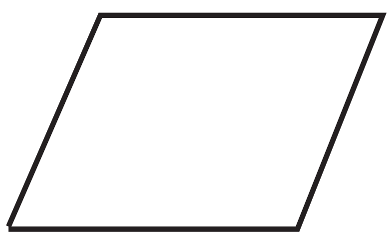
Square:



Triangle (there are many types of triangles):



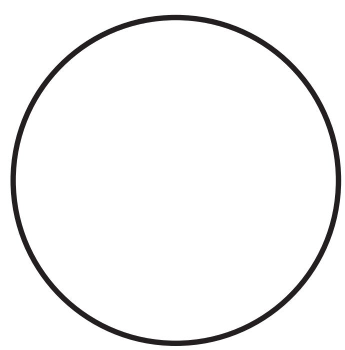
Rhombus:



Parallelogram:

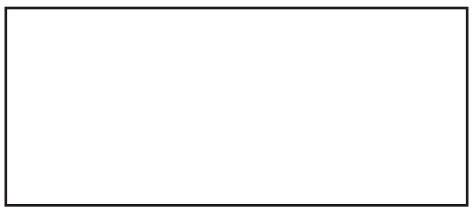
Trapezoid:

Circle:

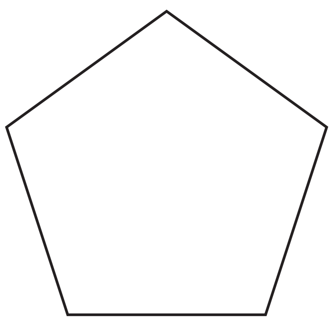


2-D Shapes

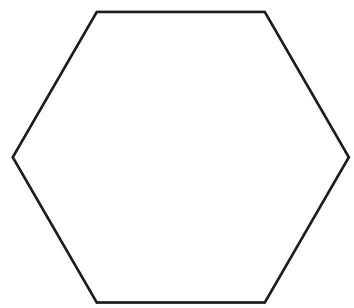
Rectangle:



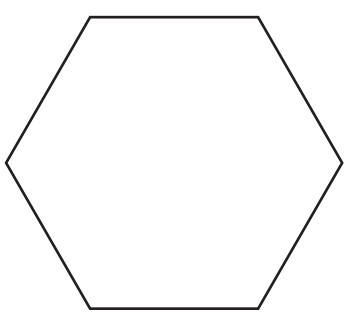
Pentagon:



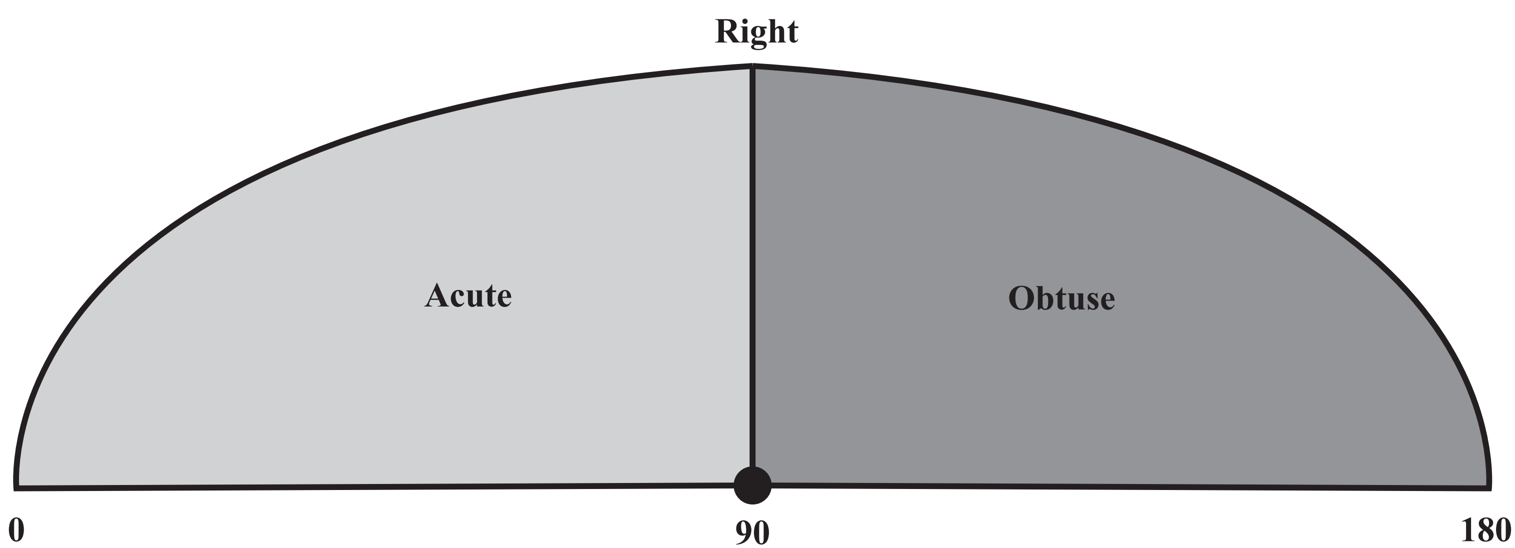
Hexagon:



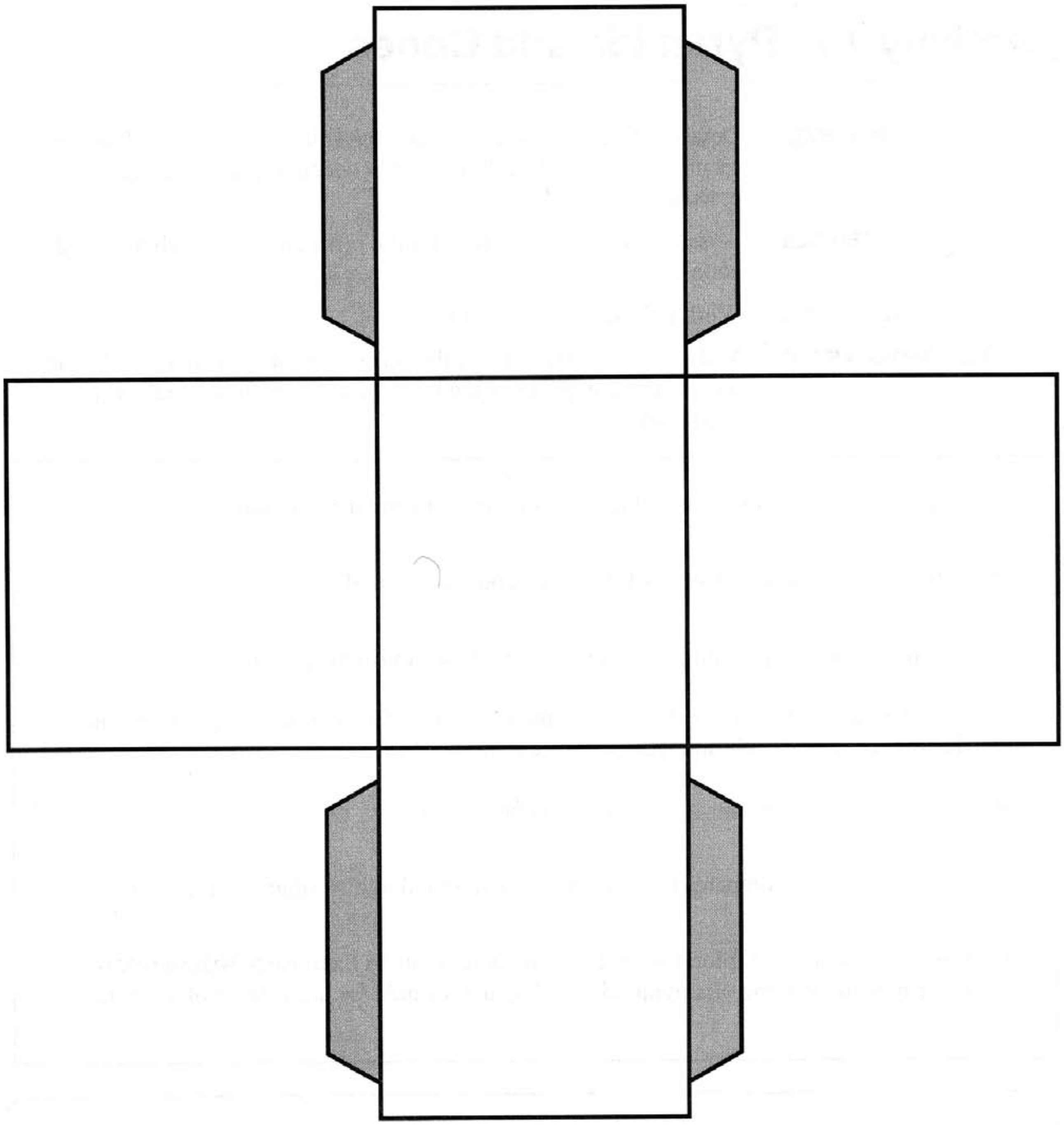
Octagon:



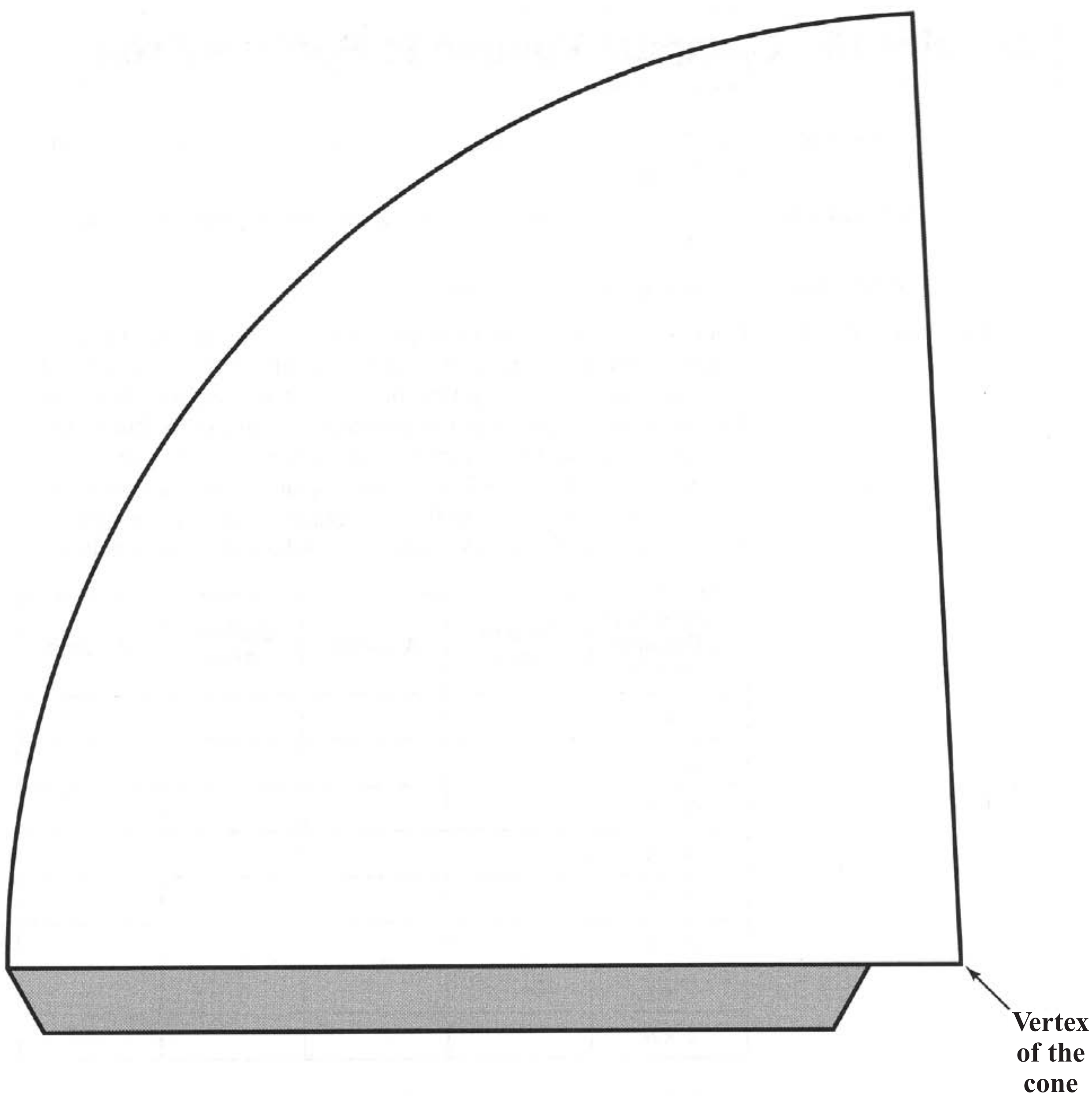
Angle Identifier



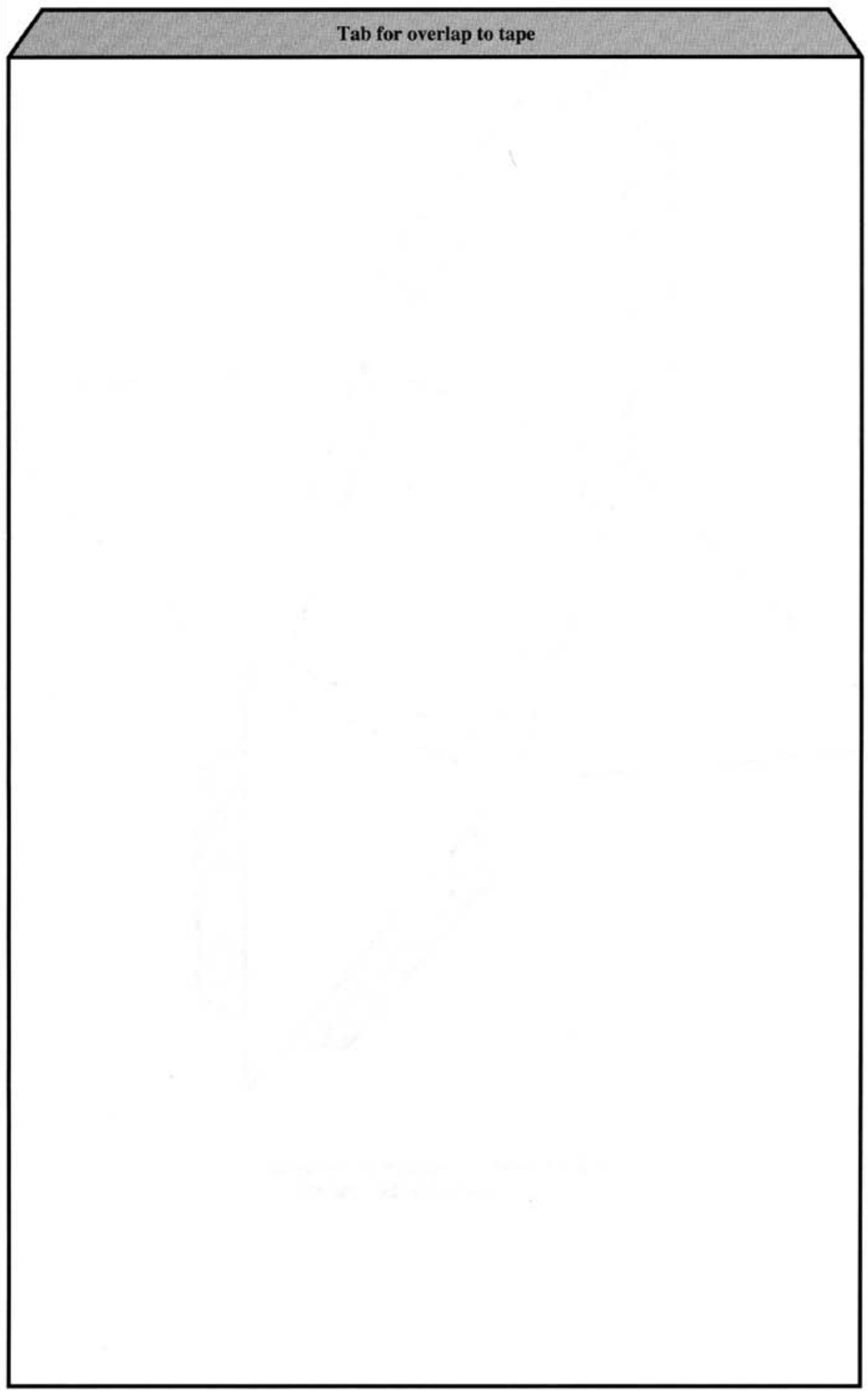
3-D Object Patterns
Net for Prism



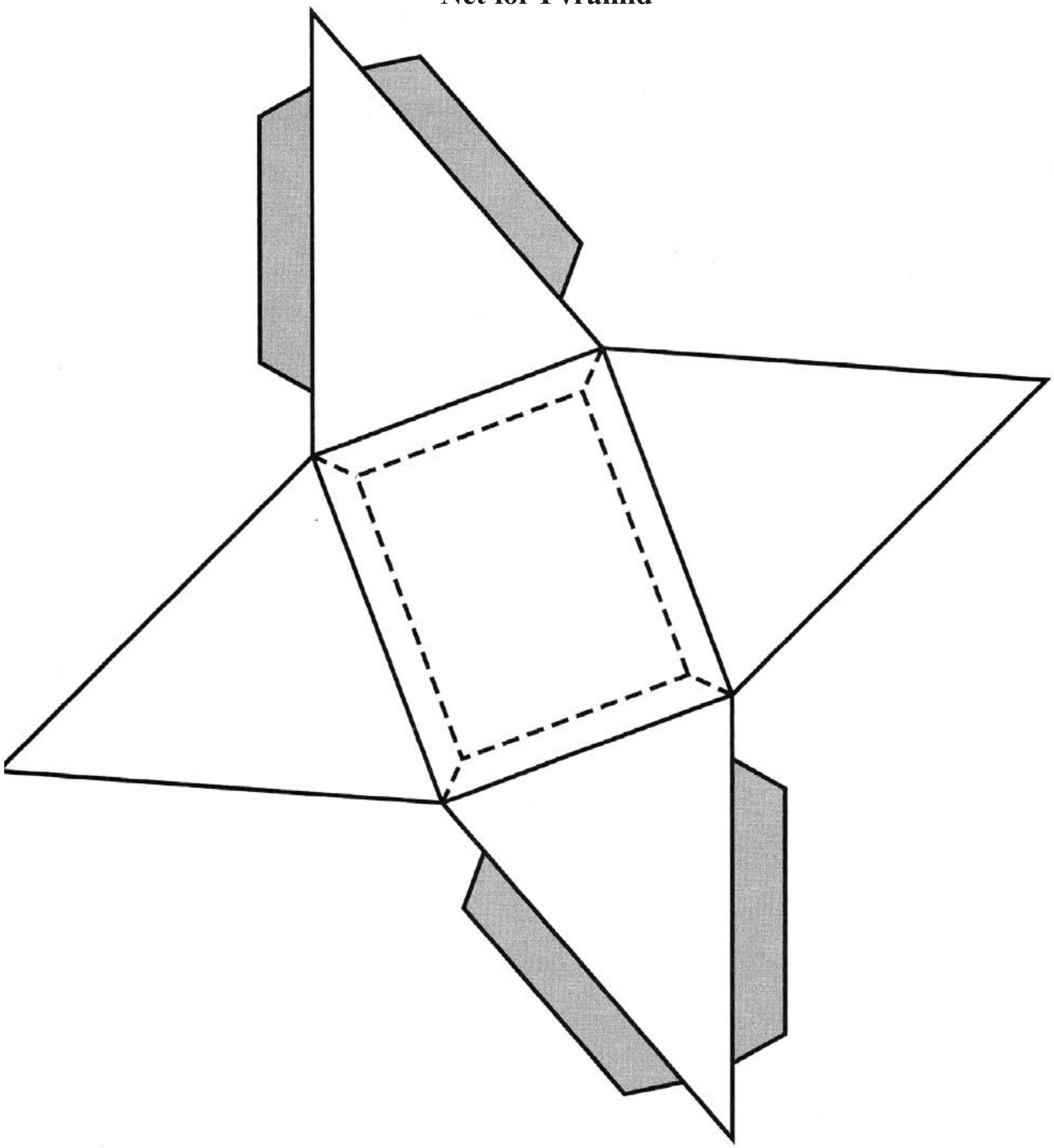
3-D Object Patterns
Net for Cone



3-D Object Patterns
Net for Cylinder



3-D Object Patterns
Net for Pyramid



Cut on dotted line to open the pyramid.
Fold the tabs inside the pyramid.

3-D Concepts

Polyhedra *(*plural for polyhedron)
Applies to shapes that contain polygons as listed for 2-D

Non-polyhedra
For this exercise, these are any 3-D shape that contains circles

Faces
Flat polygonal (shaped like polygon) surface

Edges
Place (line) where two faces come together

Vertices
(same as 2-D)
Points or corners where two edges meet

3-D Shapes

Polyhedra

Name	Picture/ Drawing	# of Faces	# of Edges	# of Vertices	Real-Life Example
Regtangular Pyramid		5	8	5	
Cube		6 (all same)	12	8	
Rectangular Prism		6	12	8	

3-D Shapes

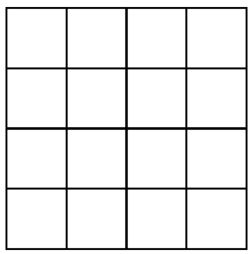
Non-Polyhedra

Name	Picture/Drawing	# of Circles	Real-Life Example
Cylinder		**2**	
Cone		**1**	
Sphere		**NA**	

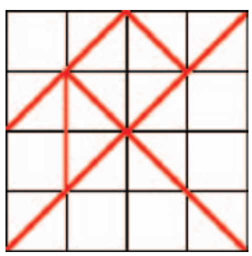
Tangrams

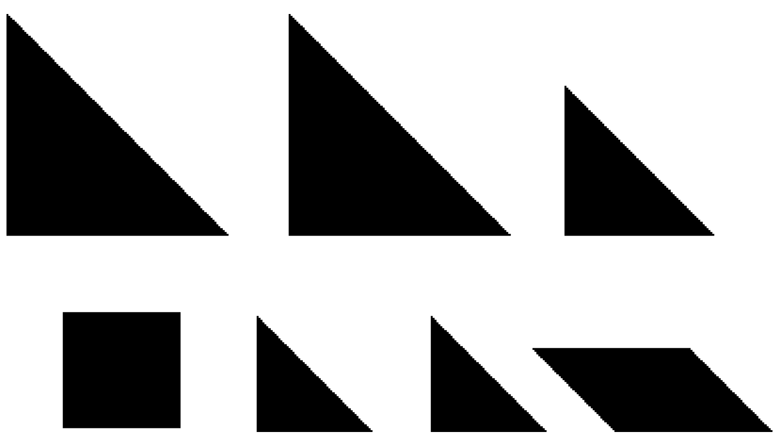
Directions for Making a Set of Tangram Pieces

1. Use large index cards.
2. Cut each card into a square (4"x4" works well).
3. Lightly mark a 4 x4 grid of squares on it, as shown in the diagram below, to act as guidelines.



4. Draw in the lines shown below and cut out along the lines.
5. This should make the 7 pieces of the tangram as shown below.





Tricky Tangrams

Use your tangrams to create the following shapes. Draw your answers
.

- Use 2 pieces to create a triangle

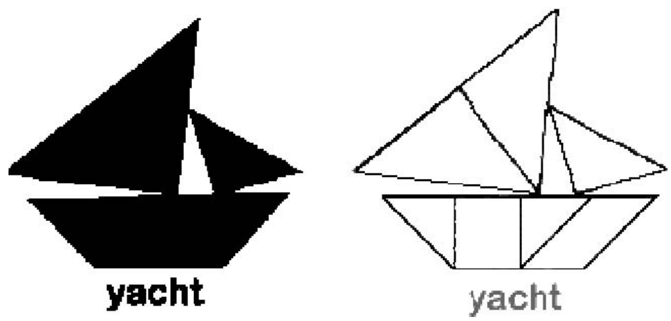
- Use 2 pieces to create a square

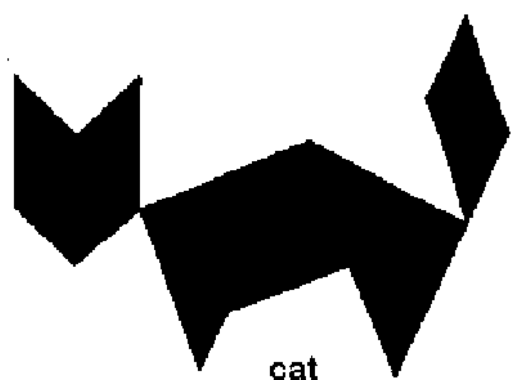
- Use at least 3 pieces to create a parallelogram

- Use 3 pieces to make a trapezoid

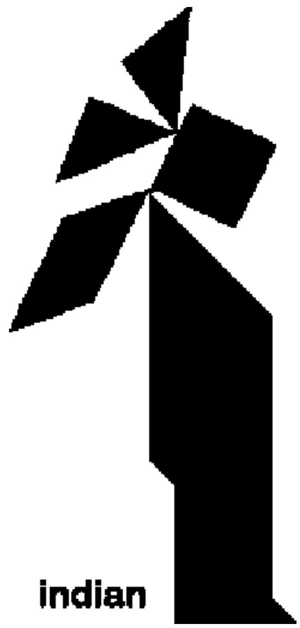
Tricky Tangrams

Use your tangrams to create the following shapes. The first gives you a model. The rest are for you to figure out!





Tricky Tangrams



Graph Paper

Congruent and Similar

Congruent shapes:

- The same angle type and measurement
- Identical length of sides

Similar shapes:

- Same shape
- All angles remain equal
- The relationship of the sides are *proportional* (the ratio of their dimensions are the same)

Rectangle 1: length of 4 and width of 2

$$4/2 = 2$$

Rectangle 2: length of 6 and width of 3

$$6/3 = 2$$

Module E
Transparencies

Module Goals
Module E: Development and Measurement

The paraeducator will:

1. Recognize and explore 2-D geometric shapes by their attributes (specific quadrilaterals, triangle, and circle; symmetry, diagonals, etc.)

2. Identify angle types (obtuse, right, acute)

3. Recognize and explore 3-D geometric shapes by their attributes (cube, sphere, cylinder, cone, prism and pyramid)

4. Solve problems using geometric relationships and spatial reasoning (e.g., congruence, similarity)

5. Use both non-standard and standard measurement for perimeter and area.

2-D Geometric Shapes

Name of Shape	Picture/ Drawing of Shape	# of sides	# of Vertices (Corners)	# and Type of Angles	# of Diagonals	Lines of Symmetry	Other

2-D Geometric Shapes

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2-D Geometric Shapes

Name of Shape	Picture/ Drawing of Shape	# of sides	# of Vertices (Corners)	# and Type of Angles	# of Diagonals	Lines of Symmetry	Other

Intersections and Vertices

Intersection

Place where two or more lines cross

Vertex
(vertices is plural)

Place where two lines (edges) intersect on a closed shape; corner

Symmetry and Diagonals

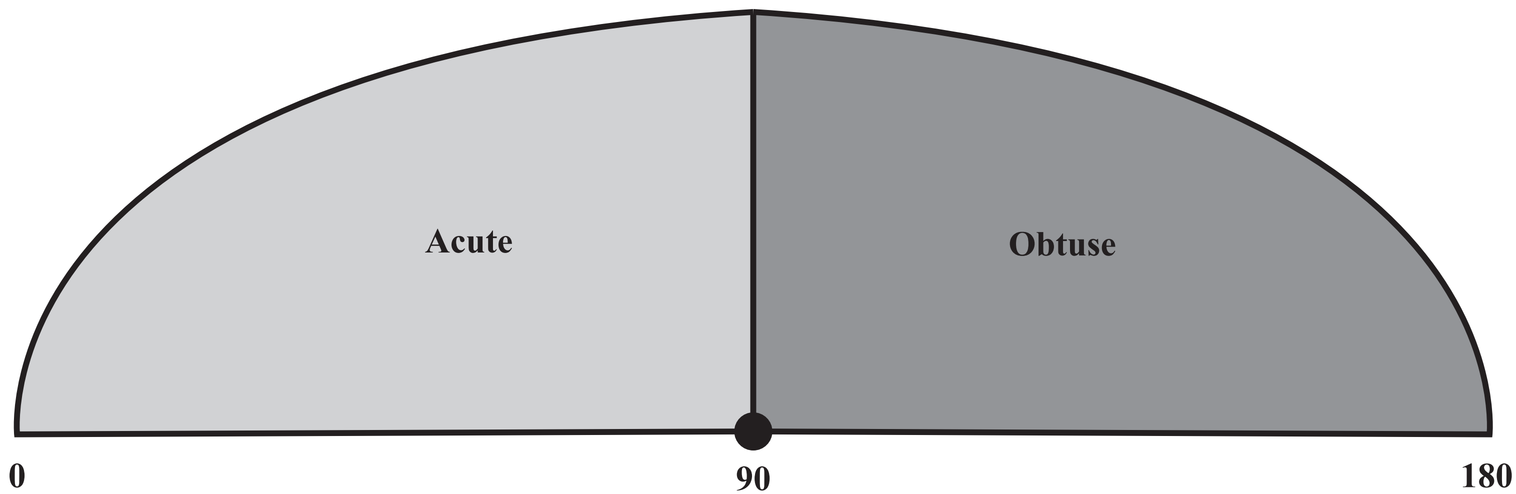
Symmetry

A line of symmetry exists if the shape can be folded exactly on itself forming two equal parts.

Diagonal

A line that goes from vertex to vertex. It cannot cut a side.

Angle Identifier



Angle

For two intersecting lines, an *angle* is the measure of rotation about the vertex to bring one line on top of the other.

Angle Concepts

- ***Degrees*** – angles are measured in degrees
- ***Acute*** – an angle with a measure between 0° and 90° (not including 90°)
- ***Right*** – "L" shape; exactly 90°
- ***Obtuse*** – an angle with a measure between 90° and 180°

Quadrilaterals

2-D shapes with four sides.

Parallel

Lines that remain the same distance apart like railroad tracks or lines on notebook paper. They never intersect (cross).

Parallelogram

Quadrilateral with two sets of parallel lines. The opposite sides are equal in length.

More Parallelograms

Rhombus

(also called a diamond)
A parallelogram with all sides equal.

Rectangle

A parallelogram with four right angles.

Square

A parallelogram with all sides equal and four right angles, or a rectangle with equal sides.

Polygon

Polygon

- *Poly* – many
- *Gon* – sided

Polygon – many-sided *closed* figure
(one continuous line that does not cross itself)

Pentagon – 5-sided figure
Hexagon – 6-sided figure
Octagon – 8-sided figure

3-D Concepts

Faces
Flat polygonal (shaped like polygon) surface

Edges
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Vertices
(same as 2-D)
Points or corners where two edges meet

Tricky Tangrams

Use your tangrams to create the following shapes:

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- Use 2 pieces to create a square
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- Use 3 pieces to make a trapezoid

Graph Paper

Congruent and Similar

Congruent shapes:
- The same angle type and measurement
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Similar shapes:
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Rectangle 1: length of 4 and width of 2

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$$6/3 = 2$$

Final Assessment

______________________ (Name)

______________________ (Date)

Final Assessment
Assisting Grades K-4 with Mathematics in the Classroom

<u>Using your notes and handouts</u> from Assisting Grades K-4 with Mathematics in the Classroom, complete the following assessment.

Module A:

1. Describe how Reys, Suydam and Lindquist (1992) define mathematics: (5 total points, I for each correct response)

Mathematics is …

1.

2.

3.

4.

5.

Module C:

Use only addition and subtraction signs to fill in the empty boxes. (20 total points, 5 points per problem)

5		7		3		1		4	=	12

20		9		1		7	=	5

Use only multiplication and division signs to fill in the empty boxes.

2		6		4		1	=	3

30		5		4		3		2	=	4

Module D:

1. **Using the following fractions, circle the number representing the "whole" and draw a box around the number representing the "part." (9 total points, 3 points per problem)**

1/4 5/8 3/5

2. **Divide the following rectangles and shade the parts as requested. (20 points, 5 points per problem)**

Divide and shade 1/2

Divide and shade 2/4

Divide and shade 2/3

Divide and shade 6/8

3. **Express the following as numbers: (4 points, 2 points per problem)**

   a. 4 hundreds + 6 tens + 9 ones = __________

   b. 8 hundreds + 0 tens + 3 ones = __________

4. **Write following as decimals: (6 total points, 2 points per problem)**

   a. 3/10 = _____________

   b. 1/4 = _____________

   c. 35/100 = _____________

5. **Write the appropriate sign in the blanks below. Use >, <, or = . (8 total points, 1 point per problem)**

   a. 192 _______ 203

   b. 14 _______ 3

   c. (1 x 4) _______ 4

   d. 5/8 _______ 3/8

   e. 2/3 _______ 2/8

   f. 0.2 _______ 0.3

   g. 0.27 _______ 0.26

   h. 0.1 _______ 0.10

Module E:
Define the following: (8 total points, 2 points per problem)

1. **What is a vertex?**

2. **How many vertices are there in a square?**

3. **How many vertices are there in a triangle?**

4. **What kind of angle is exactly 90°?**

5. Place a check mark in front of each of the following shapes that are quadrilaterals. (8 total points, 1 point per problem)

a. ______ square

b. ______ triangle

c. ______ rhombus

d. ______ hexagon

e. ______ trapezoid

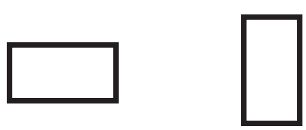
f. ______ circle

g. ______ rectangle

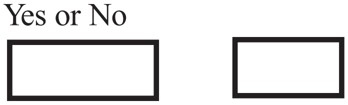
h. ______ parallelogram

6. Are the following shapes congruent? Circle Yes or No. (4 total points, 2 points per question)

a. Yes or No



b. Yes or No



Multiple Choice:

Write the letter that best fits the provided definition on the space provided. (8 total points, 4 points per problem)

1. Measurement using objects found in the environment is called:___________

    a. standard measurement
    b. non-standard measurement
    c. metric measurement
    d. English measurement

2. Comparison of an object used for measurement to a standard unit of measurement is called: ____________

    a. measuring by feet
    b. a benchmark
    c. standard measurement
    c. measuring in centimeters

Grading Rubric

Grading Rubric for Assisting Grades K-4 with Mathematics in the Classroom

This rubric includes recommendationsfor grading:
1. Participation
2. Attendance
3. Assessment
4. Assignment
5. Final grade for academy

Grades are based upon a range of possible points earned:

Participation	Attendance	Assessment	Assignment	Total points possible
0-75	0-75	0-100	0-250	0-500

A	B	C	D	Failing
500-450	449-400	399-350	349-300	299 and below

Participation: Participants can earn up to **75** points for class participation. The instructor should consider the level of participation that occurs within smaller group settings as well as in larger group opportunities.

Attendance: Participants can earn up to **75** points for full attendance. Refer to class syllabus for information regarding absences.

Assignments:
Grading is recorded and based upon a total of **250** possible points (total from both assignments). The assignment of points within each step of the assignment is left to the discretion of the instructor. Maximum points per step are assigned as follows:

Assignment Part 1: Seeing Graphs (125 points)
.
The focus of the assignment is the use of tables and graphs to represent data.

There are three steps to this assignment.

Step 1: (10 points)
Find an example of a graph in the newspaper, a magazine, the Internet, or other resource. Make a copy of that source to attach to the assignment. Accurate completion of this step of the assignment is worth a maximum of 10 points.

Step 2: (80 points)
Answer the following questions about your graph.

1. Name the type of graph from handout **Graph Types** that matches your example.
2. Give the title of your graph.

3. What kind of information is your graph measuring?
4. List the key (legend) or list the labels for the concepts on your graph being measured (vertical and horizontal direction).
5. Is it easy to read and understand? If not, provide examples of what you would do to make it easier to read.

Accurate completion of this step of the assignment is worth a maximum of 80 points.

Step 3: (35 points)
Write five questions about your graph that could be used with students. An answer key must be provided for your questions. Accurate completion of this step is worth a maximum of 35 points.

Assignment Part II: Subtraction Facts (125 points)

The focus of the assignment is to analyze subtraction facts using addition knowledge. Participants will use personal experience to answer the questions.

There are three steps to this assignment.

Step 1: (10 points)
Analyze the attached addition facts chart used in earlier activities. Explain how the chart works for addition facts. Give examples that follow your explanation.

The participant will analyze the addition facts chart used in earlier activities. The participant will explain how the chart works. Accurate completion of this step is worth a maximum of 10 points.

Step 2: (80 points)
Use what you know about the link between subtraction and addition to describe how the same chart can be used for subtraction facts. Create rules or steps that you could give to a student to explain how to use the chart to learn basic subtraction facts. Provide examples that show your rules at work.

The participant will use what he/she knows about the link between subtraction and addition to describe how the same chart can be used for subtraction. The participant will create rules to use with students explaining how to use the chart. Evidence of the inverse relationship of addition and subtraction and examples starting with the answer for each fact should be present. Accurate completion of this step is worth a maximum of 80 points. Accurate completion of this step is worth a maximum of 80 points.

Step 3: (35 points)
List 2 examples of difficulties that you have seen or experienced with students learning subtraction facts. Explain how you could use the chart to solve those student issues. Provide clear examples of your solutions.

Participants should make note of fact families (to link to addition), working with large numbers, etc. Accurate completion of this step is worth a maximum of 35 points.

Final Assessment: (100 points)
Grading is recorded and based upon a individual total of **100** possible points assigned as follows.

Student	Participation	Attendance	Assessment	Assognment	Grand Total	Assigned Grade
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						
9.						
10.						
11.						
12.						
13.						
14.						
15.						
16.						
17.						
18.						
19.						

Student	Participation	Attendance	Assessment	Assognment	Grand Total	Assigned Grade
20.						
21.						
22.						
23.						
24.						
25.						
26.						
27.						
28.						